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How did Feldstein (1985) undervalue the optimal level of social security benefits?

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How did Feldstein (1985) undervalue the optimal level of social security benefits?

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Abstract

In his seminal model (Feldstein, 1985), the government operates a social security system to counter the representative worker's myopia. (i) For a complete myope, he determined a sizable optimal tax rate (and the corresponding benefit level). (ii) For a partially shortsighted worker, he determined another optimum, which was much lower, possibly zero. Departing from Feldstein, I take into account that neither a paternalistic government nor a cautious bank tolerates long-term negative saving, and then even in (ii), the government may choose the first rather than the second optimum. Having revised it, Feldstein's model regains its place in the textbooks.

Keywords: social security, myopia, paternalism, social welfare

JEL numbers: D10, H55, J13, J14, J18, J26

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Hogyan becsülte alá Feldstein (1985) a tb-nyugdíj optimális szintjét?

Simonovits András

Összefoglaló

Nagy hatású modelljében (Feldstein, 1985) a kormányzat egy tb-nyugdíjrendszer segítségével küzd a reprezentatív egyén rövidlátása ellen. 1) Teljesen rövidlátó dolgozó esetén Feldsteinnél a társadalmilag optimális tb-nyugdíj szintje jelentősnek adódott. 2) Részlegesen rövidlátó dolgozó esetén azonban Feldstein egy másik optimumot talált, amely jelentősen kisebb, esetenként 0 volt. Feldsteinnel ellentétben nem siklom el afölött, hogy sem egy jó szándékú kormányzat, sem egy óvatos bank nem engedi meg, hogy egy dolgozó a nyugdíjjárulékot hosszú távú kölcsönből fedezze. Ekkor a kormányzat a részleges rövidlátás esetén is szükség szerint 2) helyett az 1) nyugdíjat választja: azaz a valódi optimum jóval nagyobb lehet, mint Feldsteiné. A javított Feldstein-modell visszanyeri helyét a tankönyvekben.

Kulcsszavak: tb-nyugdíj, rövidlátás, paternalizmus, társadalmi jólét

JEL-kód: D10, H55, J13, J14, J18, J26

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How did Feldstein (1985) undervalue the optimal level of social security benefits?

by

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Abstract*

In his seminal model (Feldstein, 1985), the government operates a social security system to counter the representative worker's myopia. (i) For a complete myope, he determined a sizable optimal tax rate (and the corresponding benefit level). (ii) For a partially shortsighted worker, he determined another optimum, which was much lower, possibly zero. Departing from Feldstein, I take into account that neither a paternalistic government nor a cautious bank tolerates long-term negative saving, and then even in (ii), the government may choose the first rather than the second optimum. Having revised it, Feldstein's model regains its place in the textbooks.

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I. Introduction

Since Samuelson (1958), an exploding number of papers model the interaction of social security and private saving. In a two-period overlapping generations model, the level of social security benefit is equal to the product of the per-period GDP growth factor and of the tax, while the private 'benefit' is equal to the product of the per-period interest factor and of the saving. To determine the socially optimal level of social security benefits, Feldstein (1985) used a very simple model: Choosing her saving, the representative worker maximizes a discounted lifetime utility function. Choosing the socially optimal tax rate, the paternalistic government maximizes a social welfare function, where the discount factor is higher than the individual one. Using the framework of Stackelberg-game, here the government is the leader and the individual is the follower. Though the interest factor is greater than the GDP growth factor, for very shortsighted workers, the optimal social security may partially or fully crowd out private saving.

Feldstein (1985) was the first who sought to determine the dependence of the optimal social security benefit level or equivalently, the tax rate on the individual discount factor. Using illustrative data, he came to two conclusions: (i) if the representative worker is completely shortsighted, then the socially optimal social security tax is sizable. (ii) If the representative worker is strongly but not completely shortsighted, this is not the case.

Deriving his surprising discontinuity result, however, he used a very peculiar assumption A: The worker can choose negative savings. He overlooked the fact that neither a benevolent government nor a cautious bank allows the worker to pay social security taxes from long-term credit, to be repaid from social security benefits after retirement. (In addition, he also used another 'strong' assumption B: when calculating her optimal savings, the representative worker only expects a small share of the actual benefit. As we shall see below, assumption B weakened the weight of assumption A!) Requiring nonnegative saving (assumption C), I show that even in his own numerical setup, Feldstein's first optimum may be much higher than his second and the government may choose the first rather the second.

Anticipating the details of the main results, Figure 1 displays two relative efficiency–tax rate curves—in both cases, the social welfare at any tax rate is expressed in terms of social welfare without social security: (i) curve 1 shows the relative efficiency (or consumption equivalent variation) when there is no saving; (ii) curve 2 shows the relative efficiency when saving can be negative. The true relative efficiency curve—corresponding to (C)—is equal to the maximum of the two curve. In the case described in Section 2.3, we have two local maxima: (i) at tax rate 0.4, the relative efficiency is equal to 1.064; (ii) while at tax rate 0.19, the relative efficiency is 1.026.

insert Figure 1 about here

One may doubt if there is any interest in revising an abstract model of social security from 1985, especially since Auerbach and Kotlikof (1987) opened the way to analyze detailed and calibrated general equilibrium pension models. Nevertheless, I have three reasons to share my discovery with the public:

(a) During his long and distinguished career, Martin Feldstein has been a leading partisan of cutting back the social security (from Feldstein (1974) to Feldstein (2005)), so the message of his pioneering model has exceptional importance. A separate question is whether Feldstein's pension policy is correct or not. Among others, Barr and Diamond (2008, Sections 2.2 and 3.5) convincingly argued that without taking into account the lack of complete markets and income redistribution, one cannot properly evaluate the positive role of social security.

(b) Feldstein's seminal paper serves as a starting point for many theoretical papers, beginning with Feldstein (1987) on mean-testing. To acknowledge my intellectual debt to Martin Feldstein, I refer to two papers of mine done in Feldstein's modified framework: Simonovits (2015a) on the importance of the credit constraint and Simonovits (2015b) on the choice of the socially optimal contribution rate and cap on the wage.

(c) The mistake of neglecting credit constraints is widespread in the related literature (e.g. the seminal paper by van Groezen, Leers and Meijdam (2003) and its critique by Simonovits (2015c)) on the Siamese twins of pensions and child allowances), therefore the problem deserves a treatment.

II. The model reconsidered

In my reconsideration of Feldstein's model, I try to follow closely his notations, but their clumsiness (of insisting on interest and growth rates rather than the corresponding factors) prevents me from a complete copying. To confine attention to the logical error, I do not consider the case of heterogeneous population (Feldstein, 1985, Section III). I also skip the dynamics of overlapping generations, including the problem of the first generation, the members of which received benefits without paying for it. First I shall determine the individual optimum (via private saving), then the social optimum (via tax rate). Finally I shall numerically illustrate my finding on a parameter set borrowed from Feldstein, already displayed in Figure 1.

Individual optimum

The quantities of the model are generally nonnegative real numbers, except if stated otherwise. The whole society is represented by a single individual who lives for two periods of equal lengths. (Note that the correct proportion would be 2:1, significantly reducing the optimal tax rate but this would have complicated the dynamic analysis.) In period 1, the young citizen works: she earns a unit wage, pays a tax rate θ and saves s. By definition, $0 \leq \theta < 1$ and $s < 1 - \theta$. In period 2, she receives a social security benefit b and her saving yields private pension Rs, where $R = 1 + \rho$ is the per-period interest factor.

To determine the connection between the tax rate θ and the benefit b, Feldstein introduced demography and productivity growth, represented here by the per-period population growth *factor* $N = 1 + n$ and the per-period productivity growth factor $G = 1 + g$. As is usual, $N, G, R > 1$ is assumed. Using the principle of dynamic efficiency, which also states the a priori superiority of private saving over the public one, $R > NG$ is assumed. As is known, the foregoing relation is

$$b = NG\theta. \tag{1}$$

By definition, the worker's and pensioner's consumption functions are respectively

$$c = 1 - \theta - s \quad \text{and} \quad d = b + Rs. \tag{2}$$

Determining her own saving, the worker maximizes the following discounted utility function with underestimated benefits:

$$U[s] = \ln(1 - \theta - s) + D\ln(\alpha b + Rs), \tag{3}$$

where D is the per-period discount factor ($0 \le D < 1$, λ in Feldstein, 1985) and α is the *ad hoc expected share* of her true social security benefit. Because Feldstein makes her very distrustful (assumption B), the value of the expected share is typically close or even equal to 0. Nevertheless, for greater realism, I allow for higher shares, including the so-called rational expectations ($\alpha = 1$): $0 \le \alpha \le 1$.

Taking into account (1) and (3), Feldstein's optimal saving is determined by the first-order condition

$$0 = U'[s] = \frac{-1}{1 - \theta - s} + \frac{DR}{\alpha NG\theta + Rs}. \tag{4}$$

Hence the optimal saving and the worker's corresponding consumption are equal to

$$\tilde{s}(\theta) = \frac{D(1-\theta) - \alpha NGR^{-1}\theta}{1+D} \quad \text{and} \quad \tilde{c}(\theta) = \frac{1 - \theta + \alpha NGR^{-1}\theta}{1+D}. \tag{5}$$

For the record, it is mentioned that due to $R > NG$, $\tilde{c}(\theta)$ is a decreasing function.

Note that Feldstein did *not* exclude the possibility of negative saving (no credit constraint, assumption A) and in his preferred case of $\alpha \approx 0$, for low enough tax rates, the positivity is automatically ensured. But even Feldstein sometimes considered weaker underestimation and then negative saving occurs for sensible tax rates. Since both a truly paternalistic government and a cautious bank exclude this possibility (assumption C), therefore we shall work with a modified (5): $s(\theta) = \tilde{s}(\theta)_+$, where x_+ stands for the positive part of the real x: $x_+ = x$ for $x \ge 0$ and $x_+ = 0$ for $x < 0$.

Substituting $s(\theta)$ into (2) yields the corrected consumption functions:

$$c(\theta) = 1 - \theta - s(\theta) \quad \text{and} \quad d(\theta) = NG\theta + Rs(\theta). \tag{6}$$

Government optimum

Next we turn to the government optimum. We rest satisfied with the optimal tax rate rather than the corresponding benefit level [cf. (1)]. Following Lerner (1944), Feldstein (1985) assumed a paternalistic government, which chooses the socially optimal tax rate θ to maximize the weighted sum of the current representative worker's and the corresponding pensioners' per-period utility functions:

$$V(\theta) = \ln c(\theta) + \frac{1}{N}\ln[d(\theta)/G]. \tag{7}$$

As a side remark: already Docquier (2002) noted that it would have been more logical to follow Samuelson (1958), who identified social welfare with the (undiscounted) lifetime utility

$$V(\theta) = \ln c(\theta) + \ln d(\theta). \qquad (7')$$

Note also that the current old-age consumption d/G in (7) is replaced by the future old-age consumption d in (7′), and Lerner's choice diminishes the paternalistic discount factor from 1 to $1/N < 1$.

For any given α, there generally exists a *separator* tax rate $\theta_\alpha \in [0,1]$, for which Feldstein's saving becomes zero. By (5),

$$\tilde{s}(\theta_\alpha) = 0, \quad \text{i.e.} \quad \theta_\alpha = \frac{D}{D + \alpha N G R^{-1}} \in [0,1]. \qquad (8)$$

Therefore, both consumption functions in (6) have two branches, one for $0 \le \theta \le \theta_\alpha$ and another one for $\theta_\alpha < \theta \le 1$. Except for either $D = 0$ or $\alpha = 0$ when $\theta_\alpha(0) = 0$ or $\theta_0 = 1$, both intervals are nonempty.

It will be useful to display both branches of the consumption functions:

$$c(\theta) = 1 - \theta - \tilde{s}(\theta), \qquad d(\theta) = NG\theta + R\tilde{s}(\theta) \quad \text{for} \quad 0 \le \theta \le \theta_\alpha \qquad (6a)$$

and

$$c(\theta) = 1 - \theta, \qquad d(\theta) = NG\theta \quad \text{for} \quad \theta_\alpha < \theta < 1. \qquad (6b)$$

Following Feldstein (1985, Sections I and II), we start the discussion with a degenerate case and continue with the nondegenerate case.

Complete myopia: $D = 0$**.** Then (6a) is empty, $c(\theta) = 1 - \theta$ and $d(\theta) = NG\theta$, yielding $V(\theta) = \ln(1-\theta) + N^{-1}\ln(N\theta)$. Then the first-order optimality condition is

$$V'(\theta) = \frac{-1}{1-\theta} + \frac{1}{N\theta} = 0,$$

providing the *optimal* tax rate for *completely myopes*:

$$\theta_{\mathrm{T}} = \frac{1}{1+N}, \qquad (9)$$

where T refers to the *tight* credit constraint.

We continue with the nondegenerate case, where the optimal individual saving is positive, at least for moderate social security.

Partial myopia: $0 < D < 1/N$**.** Feldstein's second optimal tax rate θ_{S} (here S refers to the *slack* credit constraint) can be determined as [cf. (6a)]
either

$$\theta_{\mathrm{S}} = 0 \quad \text{if} \quad V'(0) = \frac{c'(0)}{c(0)} + \frac{1}{N}\frac{d'(0)}{d(0)} \le 0 \qquad (10a)$$

or the unique root of

$$V'(\theta) = \frac{c'(\theta)}{c(\theta)} + \frac{1}{N}\frac{d'(\theta)}{d(\theta)} = 0 \quad \text{for} \quad 0 < \theta < \theta_\alpha. \qquad (10b)$$

We do not follow Feldstein in his drive for an explicit formula [see his (28) on p. 312] nor his separation of the *pure* private system $\theta_{\mathrm{S}} = 0$ and the *mixed* private-public system $\theta_{\mathrm{S}} > 0$. But note that in contrast to (10a), Feldstein (1985, p. 313) mentioned the possibility of nonpositive, moreover negative social security tax rate [$\theta^* \leq 0$ or $\theta^* < 0$].

It is of certain interest that Feldstein deducted θ_{T} from his θ_{S}, just inserting $D = 0$, but only for $\alpha = 0$. Note, however, that such deduction is wrong for any $\alpha > 0$, as my (5a) degenerates into $\tilde{s}(\theta) = -\alpha NGR^{-1}\theta$ rather than to 0.

Rather we concentrate on how the neglected second branch defined in $(\theta_\alpha, 1)$ influences the validity of Feldstein's result. Our starting point is that the second branch of $V(\theta)$ coincides with that studied in the case called complete myopia.

If $\theta_\alpha > \theta_{\mathrm{T}}$ holds, then the completely myopic optimum falls into the first branch, therefore θ_{T} is not an optimum: Feldstein's neglect is innocent. Using (8) and (9), this is equivalent to $D > \alpha GR^{-1}$. If $D > GR^{-1}$, then our last but one inequality holds for any $\alpha \in [0, 1]$. (Note that due to dynamic efficiency and $N > 1$, the interval $GR^{-1} < D \leq N^{-1}$ is not empty!) If $D \leq GR^{-1}$, then there exists a critical expectation share $\alpha^* = DR/G \leq 1$ and in the nonempty interval $0 \leq \alpha < \alpha^*$, Feldstein's analysis is watertight. This may explain Feldstein's preference for the unrealistically low expectation shares.

We continue the discussion with the case $\theta_\alpha \leq \theta_{\mathrm{T}}$, being equivalent to $\alpha^* \leq \alpha \leq 1$. In addition to θ_{S}, now the completely myopic rate θ_{T} is also a local optimum. Even in this second case, it is a further question which optimum is the true (global) one: the slack or the tight? If $V(\theta_{\mathrm{S}}) > V(\theta_{\mathrm{T}})$, then the optimal pure private or mixed private–public system is superior to the optimal pure social security; if $V(\theta_{\mathrm{S}}) \leq V(\theta_{\mathrm{T}})$, then vice versa. We shall see, however, that in at least one of Feldstein's numerical examples, the second case occurs and then Feldstein's neglect of the pure social security system is logically wrong.

Numerical illustration

To construct Figure 1 (displayed above) we must fill the underlying formula with numbers. I describe now the numerical data borrowed from Feldstein. Working with period length of 30 years: the cumulated factors are respectively equal to $D = 0.05 = 1/(1 + \mathbf{d})^{30}$ (due to annual discount rate $\mathbf{d} = 0.105$), $N = (1 + 0.014)^{30} = 1.52$, $G = (1 + 0.022)^{30} = 1.921$ and $R = (1 + 0.08)^{30} = 10.063$ (Feldstein, 1985, pp. 307 and 313–314). Note also that now $\alpha^* = DR/G = 0.05 \times 10.063/1.921 = 0.262$ is the critical value, well below $\alpha = 0.5$. Here $\theta_{\mathrm{S}} = 0.19$, while $\theta_{\mathrm{T}} = 0.4$.

To compare the social welfare's values provided by the two local optima, we shall use the relative efficiency of both optima. For $D > 0$, the relative efficiency of θ with respect to $\theta = 0$ is defined as a positive number $\varepsilon(\theta)$, with which multiplying the unitary wage and the corresponding benefit level, the social welfare at $\theta = 0$ becomes equal to the social welfare at θ without changing the unitary wage and the benefit. In formula: $V[\varepsilon, 0] = V[1, \theta]$, i.e.

$$\varepsilon = \exp\{(V[1, \theta] - V[1, 0])/2\} = \exp\{(V(\theta) - V(0))/2\}.$$

Then

$$\varepsilon(\theta_{\mathrm{S}}) = 1.026 \quad \text{and} \quad \varepsilon(\theta_{\mathrm{T}}) = 1.064.$$

It is difficult to resist the temptation to present the results for the neglected rational expectations, i.e. $\alpha = 1$. Then θ_S drops to 0 while θ_T remains 0.4. The difference between the relative efficiencies of T and S-optima grows from 0.038 to 0.064!

To round the picture, for $\alpha = 0$, $\theta_S = 0.28$ and $\varepsilon_S = 1.136 > 1.117 = \varepsilon_T$, thus in Feldstein's favorite case, the mixed system is indeed welfare superior to the pure social security.

In words: the relative efficiency of Feldstein's local optimum is frequently if not always significantly lower than that of the global optimum. Feldstein's error in undervaluation of the socially optimal social security benefits is demonstrated. Of course, it would be useful to give a general analytical condition on the superiority of this or that solution (cf. Simonovits, 2015a).

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Figure 1. Two welfare curves