MAGYAR TUDOMÁNYOS AKADÉMIA
Közgazdaság- és Regionális Tudományi Kutatóközpont

MTA KRTK KTI

Centre for Economic and Regional Studies
HUNGARIAN ACADEMY OF SCIENCES

MŰHELYTANULMÁNYOK DISCUSSION PAPERS

MT-DP – 2016/33

Negative correlation between retirement age and contribution length?

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Discussion papers
MT-DP – 2016/33

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November 2016

ISBN 978-615-5594-74-8
ISSN 1785 377X

Negative correlation between retirement age and contribution length?

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Abstract

Though never stated explicitly, there is a hidden hypothesis that in a normal pension system, the retirement age and the contribution length are strongly and positively correlated. We compare the time paths of male and female correlation coefficients in Austria, Hungary, Germany and Sweden for several years and categories; and obtain a mixed picture. Hungary stands out with its strong negative correlation but the remaining three countries cannot boast with strongly positive correlation, either. Further work is needed to understand the significance of our findings but they signal some problems with these systems: heterogeneously fragmented careers and unfair benefit rules.

Keywords: public pension system, length of employment, fragmented careers, retirement age

JEL classification: H55, J26, J64

Acknowledgements:
We express our indebtedness for Yali Liu for constructive help.
András Simonovits' research was supported by the Hungarian Research Fund OTKA 108668.

Negatív korreláció a nyugdíjba vonulási kor és a szolgálati idő között?

Erik Granseth – Wolfgang Keck – Wolfgang Nagl – Simonovits András – Tir Melinda

Összefoglaló

Bár soha sem mondták ki explicite, mégis létezik egy rejtett hipotézis: normális nyugdíjrendszerekben a nyugdíjba vonulási kor és a szolgálati idő hossza erősen és pozitívan korrelál. Ebben a dolgozatban összehasonlítjuk az osztrák, a magyar, a német és a svéd mutatókat férfiakra és nőkre, több évre és kategóriára lebontva, és vegyes képet kapunk. Magyarország kiválik erősen negatív korrelációjával, de a másik három ország sem dicsekedhet erős pozitív korrelációval. További munkára van szükség, hogy megértsük megfigyeléseink jelentőségét, de annyit már most elmondhatunk, hogy problémákra utalnak: heterogén töredezettségű munkaviszonyra és méltánytalan járadékokra

Tárgyszavak: tb-nyugdíjrendszer, szolgálati idő, töredezett munkaviszony, nyugdíjba vonulási kor

JEL kódok: H55, J26, J64

1. INTRODUCTION

In aging populations, the proportional rise of the average (or effective) retirement age is a key requirement of any pension system. There is a vast literature concerning the impact of the retirement rules on the retirement age (e.g. Gustman and Steinmeier, 1986; Stock and Wise, 1990 and Gruber and Wise, eds. 2007). In well-designed pensions systems, the pension benefit is an increasing function of the retirement age and of the lifetime contribution in a wide interval of these two variables. Since we shall not consider wages, therefore the average lifetime contribution will be replaced by the *contribution length*.[1] In the simplest setting (neglecting growth and interest), the so-called actuarially fair pension benefit is proportional to the length of contributions and inversely proportional to the remaining life expectancy, which in turn is an almost linear decreasing function of the retirement age in the relevant interval.[2]

Though never stated explicitly, there is a hidden hypothesis that in a normal pension system, *the retirement age and the contribution length are strongly and positively correlated*. In fact, a hidden but invalid assumption lies behind the hypothesis on correlation: the employment career is generally continuous (non-fragmented). If a typical employment were continuous, then the contribution length would simply be equal to the retirement age less the age starting to work, say 20 years.[3] Then delaying retirement by one year would increase the employment length by one year. Furthermore, if the degree of fragmentation were positive but uniform, the correlation coefficient would remain the same, close to +1.

But in countries in transition in general and in Hungary in particular, a large share of workers has fragmented employment careers, while others still have continuous careers (e.g. Augusztinovics and Köllő, 2008 and 2009). Similarly heterogeneous fragmentation can be observed in mature market economies (e.g. Chan and Stevens, 2004; Etgeton, 2016 and Manoli and Weber, 2016). Therefore the above deduction on the strongly positive correlation coefficient is not valid.

In an Appendix we present two simple pension models. In Model 1, with rigidly defined retirement age, the corresponding correlation is negative. In Model 2, with flexible (variable) retirement age, the correlation is positive. Without going into the details of a country's

[1] We shall use alternatively also length of employment/contribution or years of contributions.
[2] Another definition of fairness: the social security wealth is invariant to the retirement age.
[3] The common starting point is a good approximation for some countries, like Hungary, where the years of higher education (at least obtained before 1998) count as years of contribution.

pension system, we conjecture that the less rigid the system, or equivalently, the more flexible the retirement age, the higher the algebraic value of the correlation coefficient.

Having analyzed the gainers and losers of the Hungarian retirement rules, Czeglédi, Simonovits, Szabó and Tir (2016) observed time-varying but strongly negative correlation coefficients, especially for females. For example, the existence of peculiar pension rules (like seniority pensions) and uncertainty concerning pension policy may imply a paradoxical behavior: the longer the employment length, the earlier one retires.

Following this pilot study on the Hungarian correlation coefficients, in the present paper we make a limited international comparison for four countries to see their time development; and try to make judgment on the flexibility of the foregoing pension system through its correlation coefficient. Obviously, the results may depend on the sex of the group (females have more fragmented careers and frequently retire earlier than males, coordinating their decisions with their typically older husbands) and the width of the pensioner category (all or old-age pensioners, contribution length is arbitrary or greater than a relatively high value, etc.). As a rule, the signed correlation coefficients are almost always higher for males than for females but their differences vary across countries.

Without going into details, note that each of the foregoing countries has had a dominant public pension system, with a relatively high replacement rate.[4] The strength of the link between lifetime contribution and lifetime benefits varied across time and space, but they were definitely positive. We have obtained the following empirical results.

1. (a) Between 2003 and 2010, in Hungary, for those retiring above age 54, the correlation coefficient was at most –0.3 and sometimes reaching –0.7. (b) By requiring minimum 20 years of employment, the correlation rose a little bit but not much. (c) Finally, confining attention to those retiring at normal retirement age, the correlation coefficient was always higher than –0.3 and frequently close to 0.

2. The situation in Austria has been somewhat different. (a) Unrestricted male and female correlation coefficients oscillated between 0.1 and –0.5, respectively. (b) Requiring 20 years of employment slightly lifts the correlations. (c) Confining attention to old-age retirement, both correlation coefficients are even lower, and oscillate between –0.5 and –0.7, respectively.
(d) Requesting at least 20 years and old-age retirement, the coefficients slightly rose but not much.

3. Turning to Germany, (a) requesting a minimum length of 15 years, the correlation coefficients oscillated between –0.8 and –0.2, sometimes the female indicator being higher

[4] Rather high in Austria and Hungary, not so high in Germany and Sweden, but still much higher than in the public pillar of the UK and the US.

than its male counterpart. (b) Requiring at least 35 years of contributions (to be called critical length), the coefficients rose close to 0: for males, a little bit above 0, for females starting with negative, and continuing with positive values.

4. Last but not least, the Swedish system seems to be the fairest. (a) In the unrestricted version, both correlation coefficients were close to each other and to 0. (b) Requiring at least 30 years of service, the coefficients were higher but close to 0. (c) Sorting out those who have less than 30 contributive years, we obtain a declining curve, reaching –0.3 for females in 2015.

In summary, Hungary stands out with its strong negative correlation but the remaining three countries cannot boast with strongly positive correlation, either. Further work is needed to understand the significance of our findings but they signal some problems with these pension systems.

The structure of the paper is as follows. Section 2 contains the framework. Sections 3, 4, 5 and 6 present the country studies for Hungary, Austria, Germany and Sweden, respectively. Section 7 concludes. The paper closes with a theoretical Appendix.

2. ANALYTICAL FRAMEWORK

Before presenting the country studies, it is worth constructing an analytical framework. Consider a country with a cohort born in year t. (In fact, in most countries, one should also distinguish females and males.) Index the cohort members with $i = 1, 2, \ldots, n$, denote the retirement age, the corresponding contribution length, and lifetime gross wage by R_i, S_i and w_i, respectively. Obviously, the annual benefit b_i depends on R_i, S_i and w_i: $b_i = b(R_i, S_i, w_i)$, typically increasing in all the three variables. We call a system *flexible* if the elasticities of the benefit to the three variables are positive and far from being zero in a large feasibility domain (e.g. $S \geq 20$ and $R > 60$). In contrast, we call a system *lenient* if the second elasticity is zero for long enough contribution period (say 35-40 years); and *rigid* if the benefit is not defined in the subdomain of the feasibility domain ($S < 40$ and $R < 63$). In a lot of countries, the normal (statutory) retirement age (R^*) and the critical contribution length (S_o) play an additional role; (a) in a flexible (variable) system, retiring before/after reaching R^* implies a deduction/credit and (b) if a worker has a contribution length of at least S_o, she is entitled to early retirement without deduction.

The standard theoretical literature typically assumes a simple relation between the retirement age R_i and the contribution length S_i, e.g. $S_i = R_i - L$. Note, however, the basic insight of the relevant literature (e.g. Czeglédi et al., 2016) that there is no such a relation. As a first approximation, we may assume that a worker considers her/his retirement age R_i, knowing that her/his length of employment is determined by $S_i = \Phi_i (R_i)$, where $\Phi_i (\cdot)$ is an increasing function, i=1, 2,..., n. In the simplest case, this function is affine: $S_i = \varphi_i (R_i - L)$, where $1-\varphi_i$ is the *degree of fragmentation* of employee i's career. Understanding this connection and the benefit function $b(R, S, w)$, the employee chooses her/his retirement age.

To define the correlation coefficient between the retirement age and the contribution length, we need the expected retirement age and the expected employment length, respectively:

$\mathbf{E}R = (1/n)\sum_i R_i$ and $\mathbf{E}S = (1/n)\sum_i S_i$

and their variances:

$\mathbf{D}^2R = \mathbf{E}(R - \mathbf{E}R)^2$ and $\mathbf{D}^2S = \mathbf{E}(S - \mathbf{E}S)^2$.

Finally we define the *correlation coefficient* between R and S:

$\rho(R, S) = \mathrm{E}((R - \mathbf{E}R)(S - \mathbf{E}S))/(\mathbf{D}R\, \mathbf{D}S)$ if $\mathbf{D}R > 0$ and $\mathbf{D}S > 0$.

As is known, $-1 \le \rho(R, S) \le 1$, and the equalities hold if and only if $S = AR + B$, with $A < 0$ and $A > 0$, respectively. Note that if all the degrees of fragmentation φ_is were close to each other, then $\rho(R, S) \approx 1$ but this is not the case.

At the end of the Section, we repeat our starting hypothesis: *the more flexible the pension system, the greater the algebraic value of the correlation coefficient*. In our Appendix, we shall consider two extreme examples and illustrate this hypothesis numerically.

3. CORRELATIONS IN HUNGARY

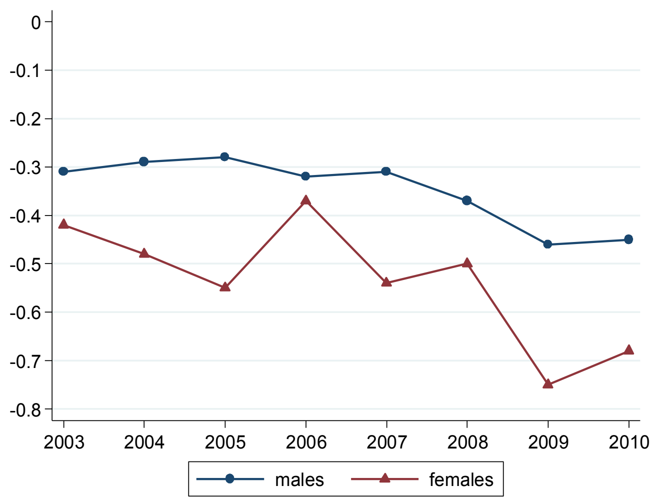
Apart from an aborted trial to carve out a second, mandatory funded private pillar from the Hungarian public pension system between 1998 and 2010 (see, Simonovits, 2011), the Hungarian mandatory pension system has been basically unfunded. The progressivity has practically been phased-out. Between 2003 and 2010 (and also later), the retirement rules frequently changed in Hungary (cf. Czeglédi et al., 2016 and Freudenberg, Berki and Reiff, 2016). Most notably, the female normal retirement age (at which workers could retire without any limitation) was raised from 55 (in 1995) to 59 in 2003, 60 in 2005, 61 in 2007 and 62 in 2009. Male's normal retirement age was already 62 in 2001. One may find the unified normal retirement age low but it is already 63 and will reach 65 by 2022.Any rise in the normal retirement age is only meaningful if it punishes those who retire below it. In principle, this was the case, but in practice, there were a lot of transient rules which almost eliminated the deduction for early retirement, at least until 2010.

The following three figures show that, at least in Hungary, between 2003 and 2010, the foregoing correlation was negative, sometimes very strongly negative. (For the data used, see Czeglédi et al., 2016). The reason may be simple: those who acquired the critical contribution period, retired immediately without any deduction; the others had to work until reaching normal retirement age.

Figure 3.1 concerns all workers who retired above 54. For them, the foregoing correlation was only −0.3 in 2003, but it dropped to −0.45 by 2007. For females, the situation is even more paradox: the indicator started from −0.4 and dropped to −0.7 by 2010. The local female minima at odd years were caused by the rise in the normal retirement age by one year in these years.

Figure 3.1.

Correlation coefficient for those retiring above 54, between 2003 and 2010, Hungary

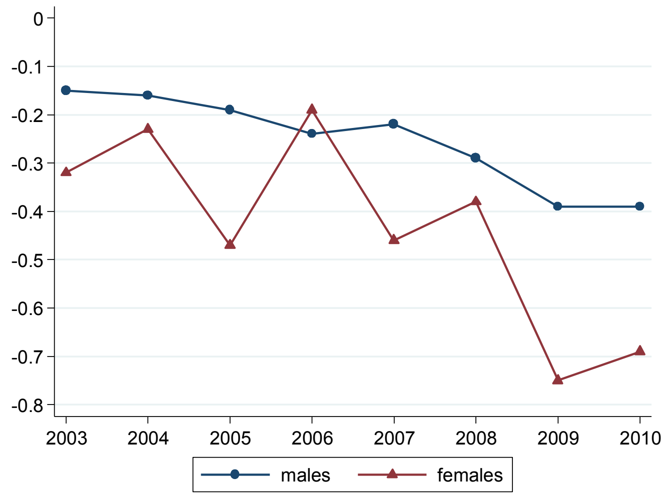


Source: Czeglédi et al. (2016)

To exclude outliers, we confine our attention to those who had at least 20 years of employment (the recent minimum value). Similarly to Figure 3.1, Figure 3.2 also reports negative and time-decreasing correlation.

Figure 3.2.

Correlation coefficient for those retiring above 54, with minimum 20 years of employment, between 2003 and 2010, Hungary

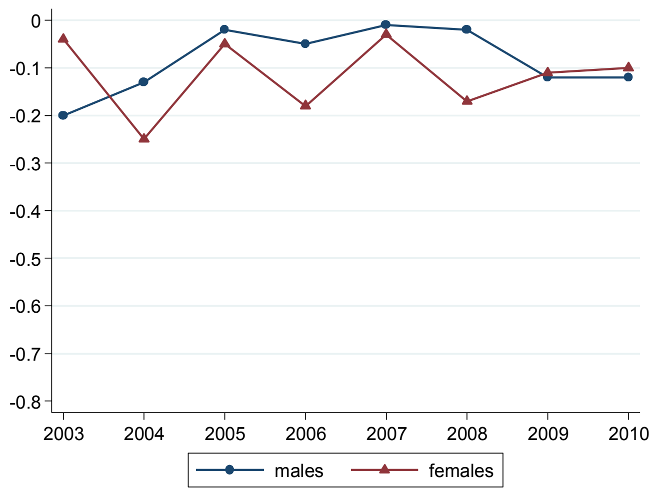


Source: Czeglédi et al. (2016)

Further delimiting the analysis, we only consider those who retired at or above the normal retirement age. Figure 3.3 still reports negative correlation but with low absolute values.

Figure 3.3.

Correlation coefficient for those retiring at or above the normal retirement age, between 2003 and 2010, Hungary



Source: Czeglédi et al. (2016)

4. CORRELATIONS IN AUSTRIA

The public pension system (the dominant pillar) in Austria is a defined benefit pay-as-you-go scheme (cf. Manoli and Weber, 2016). Since the General Pension Act of 2004, to calculate pension benefits the complete employment history is to be taken into account. Before this reform the replacement rate increased with the years of employment but only the 15 years with the highest income were taken into account to calculate the monthly pension. For entitlements before 2004, grandfathering applies, so there is a long transition period before the new system fully applies. Note that today's retirees are not or only a little affected by the new system. The normal (statutory) retirement age is currently 65 for males and 60 for females, but the latter will increase up to 65 between the years

Until 2017 it is still possible to retire before the statutory retirement age without deductions if the individual contribution period is long enough (35 years) and in the past this was an often used pathway into retirement, especially by males. There are mainly three ways to choose early retirement with deduction: early retirement due to a long insurance history, pension due to hard labor and a corridor pension. For all three forms of early retirement different but long insurance periods are required.[5] People who are not able to work anymore are covered via invalidity pensions. Up to 2014 it was part of the public pension system, now it is part of the health insurance system.

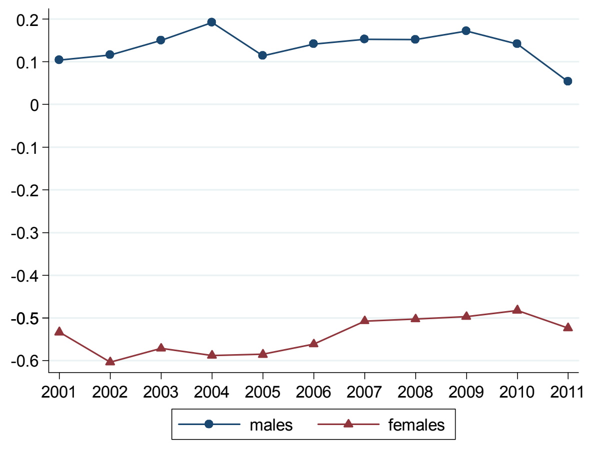
Using the IHS Microsimulation Model for Retirement Behavior in Austria (IREA) we calculate correlations between the retirement age and the individual contribution period.[6] To ensure complete insurance histories in IREA, two very detailed micro datasets are merged: The Condensed Insurance Periods and Pension Calculation dataset[7] and the Austrian Labour Market Database.[8] Since 1972 the AMDB has provided a total census of all employed persons in Austria, but our primary source of information is the VVP. The VVP is a 50% sample of all new retirees between the years 2001 and 2011 and contains all pension relevant information (insurance periods, assessment base, individual pension entitlement, date of retirement etc.).

The overall correlations between the retirement age and the contribution length are quite different for males and females in Austria. For all new male retirees over 54 between 2001 and 2011, the annual correlation was positive, for all female new retirees, it was negative (see Figure 4.1).

[5] For detailed information see Federal Ministry of Finance (2012) and Hanappi et al. (2012).
[6] For a detailed description of the IHS Microsimulation Model for Retirement Behavior in Austria (IREA) see Hofer et al. (2012).
[7] *Verdichteter Verlauf Pensionen,* VVP
[8] *Arbeitsmarktdatenbank,* (AMDB

Figure 4.1.

Correlation coefficient for males and females retiring above 54, Austria

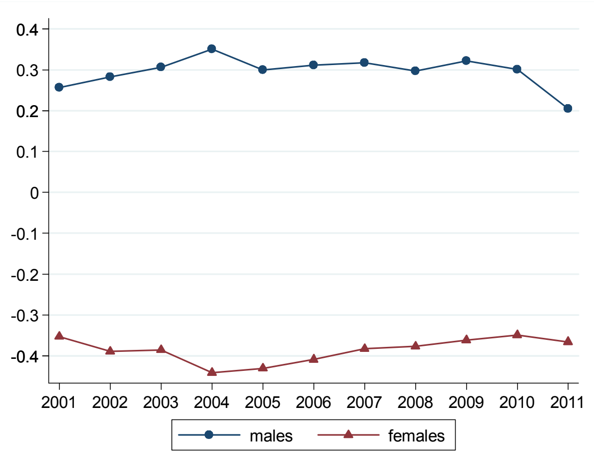


Source: VVP and AMDB 2001-2011.

The explanation is simple: males with more years of contribution retire later and females with more years of contribution retire earlier. When we focus on people with at least 20 years in employment the positive correlation for males becomes even more pronounced, whereas the absolute value of the negative correlation for females is reduced (see Figure 4.2).

Figure 4.2.

Correlation coefficients for retiring above 54, with minimum 20 years of employment, Austria



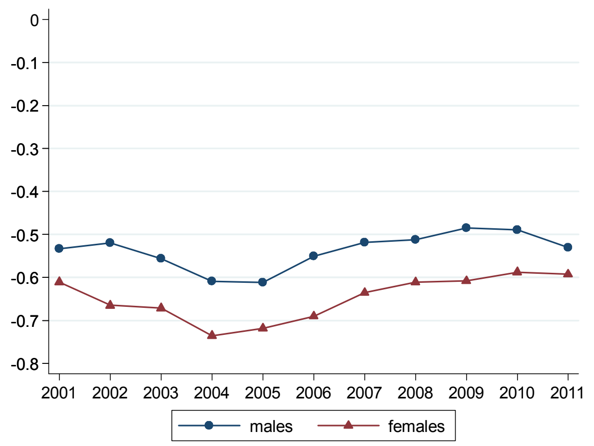
Source: VVP and AMDB 2001-2011.

The strong gender difference is most probably driven by invalidity pensions. Excluding disabled pensioners, the correlations for males and females are quite similar (see Figures 4.3 and 4.4). The negative correlation for old-age male and female retirees is what one would most probably expect because there are, as mentioned above, a variety of pathways into early retirement in Austria. For these pathways, it is mandatory to have long insurance periods.

The lower negative correlation for females might occur because of the lower statutory retirement age for females in Austria. The restriction of a minimum of 20 years of employment reduces negative correlation a little (see Figure 4.4).

Figure 4.3.

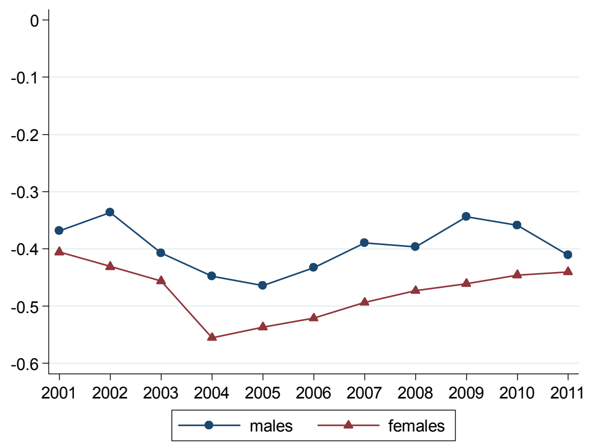
Correlation coefficients for retiring above 54, only old age pension, Austria



Source: VVP and AMDB 2001-2011

Figure 4.4.

Correlation coefficient for retiring above 54, with minimum 20 years of employment, only old age pensions, Austria



Source: VVP and AMDB 2001-2011.

5. CORRELATIONS IN GERMANY

In Germany the public pension system is the dominant pillar (cf. Werding, 2016). Contrary to Hungary or Austria, the benefit has always been proportional to lifetime average earnings. The minimum contribution length is equal to five years. The shortest contribution length to early retirement scheme which is accessible to females and unemployed persons is equal to 15 years. If a person contributed for 35 years, a general access to an early retirement scheme is provided. Insurance records include not only months with pension contributions due to employment, short-term unemployment, and care giving but also non-contributory periods which are acknowledged for the pension calculation: schooling times, long-term unemployment, child-rearing, sickness etc.

If the empirical test of the hypothesis should be more than a reflection of the accessibility of the early retirement schemes, the analyses have to be restricted to two subsamples of retirees. The larger group formed by those retirees who have at least a contribution length of 15 years (98% of all retirees both for females and males). The smaller group formed by those retirees who have in general access to an early retirement scheme due to 35 years of insurance records (75% of males and around 50% of females).[9]

Reiterating, in the German pension system, the benefit amount has always been strongly linked not only to the contribution length, but also to the retirement age and the lifetime wage. Major pension reforms in Germany since 1992 have even tightened this relationship. On the one hand, actuarial deductions have only been introduced between 1998 and 2004, and reforms took effect in the two schemes with rather lenient eligibility criteria in terms of the 15 years of contributions. Here the maximum deduction increased from zero to 18 percent of the pension claim if a person retires five years before the statutory retirement age. A second strand of the reform raised the minimum retirement age for the statutory pension age as well as for the early retirement schemes. Earliest retirement age was increased from 60 to 63 years from 2005 to 2011 (except for severely disabled persons) and statutory retirement age will increase from 65 in 2012 up to 67 in 2029. Finally the two early retirement schemes with a short waiting period of 15 years of contribution were abolished for those born after 1952 which mean that from 2017 onwards these schemes will not exist anymore.

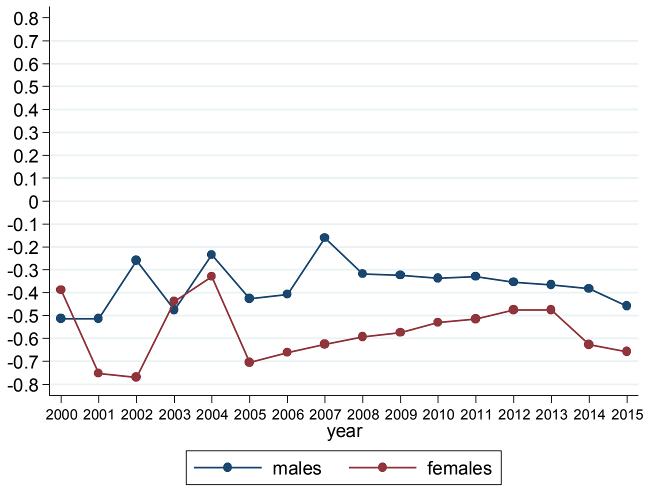
A backlash to these dominant reforms in the last two decades happened in 2012 (and again reformed in 2014) with a new early retirement scheme for very long insured persons with a waiting period of 45 years. Since 2014 this scheme has been allowing an early retirement with the age of 63 without deductions for some birth cohorts.

[9] Unfortunately, the data refer to the years of contributions rather than of employment. Thus there is a different classification than the analyses from Austria and Hungary. However, in Germany, the vast majority of contributions records are related to employment.

Figure 5.1 shows the correlation between years of contributions and retirement age for all new retirees having achieved a minimum of 15 years of contribution. The results are pretty much a reflection of the pension reforms. Between 2000 and 2004 pension deduction were introduced for early retirement and therefore the decisions on retirement depended at that time to a substantial part in optimizing the pension benefit by using the (closing) windows of opportunity. The peak in 2007 for males may have something to do with a labor market reform. Until 2007 unemployed persons could retire at the age of 60 if they have previously been unemployed for at least 2 years. Since 2008 this option has been closed. Females are not so much affected because they have still the opportunity to retire at the age of 60 in a pension scheme accessible for them only. Between 2008 and 2013 in particular the correlation coefficient for females and males converged mainly because of the growing restriction for early retirement for females. The break in 2014 has probably to do with the reform of the early retirement scheme for very long insured persons.

Figure 5.1.

Correlation coefficient for at least 15 years of contribution, Germany

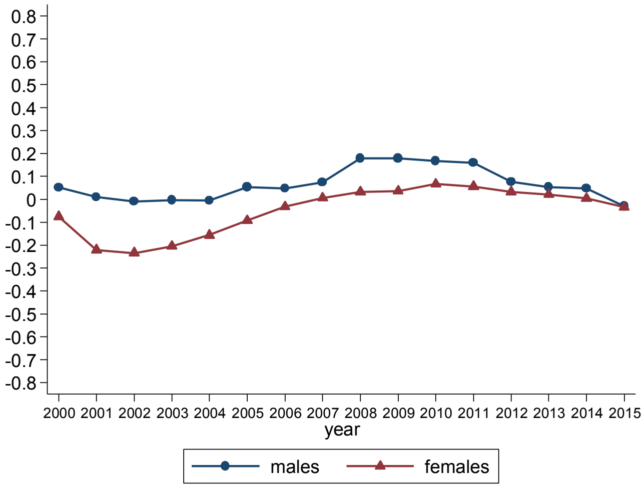


Source: Deutsche Rentenversicherung Bund (German Pension Insurance): Rentenzugang 2000-2015

The developments over time are smoother when considering only retirees with longer insurance periods (Figure 5.2). This subsample is not so much affected by the ongoing pension reforms. Pension deductions for the early retirement scheme for long-term insured persons were already introduced in 1998. Of course, this group could make use of different early retirement schemes, also those with more lenient eligibility conditions. But at the same time there is an increasing share of individuals who continuously stay in the labor market in later life, and who do not perceive job pressures to retire earlier.

Figure 5.2.

Correlation coefficient with at least 35 years of insurance, Germany



Source: Deutsche Rentenversicherung Bund (German Pension Insurance): Rentenzugang 2000-2015

6. CORRELATIONS IN SWEDEN

The Swedish mandatory pension system has two pillars: the first pillar (the dominant) is an unfunded public system, while the second pillar (subordinate) is a mandatory private funded system (cf. Palmer, 2000 and Sundén, 2006), Sweden is still in transition from a defined benefit system (DB) to a notional defined contribution system (NDC, inkomstpension). In the old system, 30 years of contribution were needed for full qualification. Because the new pension system is a notional defined contribution (NDC) system, there is a direct linkage between the number of contributing years (pension contribution), the retirement age and the pension. If the pension level is beneath a certain threshold, there is a top-up to a minimum level by the guaranteed pension (garantipension) and/or housing benefit. In Sweden, the minimal retirement age is 61. The guaranteed pension is only paid after the 65th birthday.

Figure 6.1 shows that there is almost no correlation between the retirement age and the number of working years for the period between 2003 and 2013. Since 2013 there is a weak negative correlation. The reason for the drop is a selection effect. Cohorts 1949 and 1950 became 65 years old in 2014 and 2015, and if they were included, the correlation coefficient for those years would be around zero for both sexes.[10]

[10] One reason for why the correlation coefficient is close to zero is because a majority retires when they are 65 years old. Cohorts 1949 and 1950 were excluded since only the early retirees would have had time to retire in the period studied.

Figure 6.1.

Correlation coefficient for those born 1938–1948, Sweden

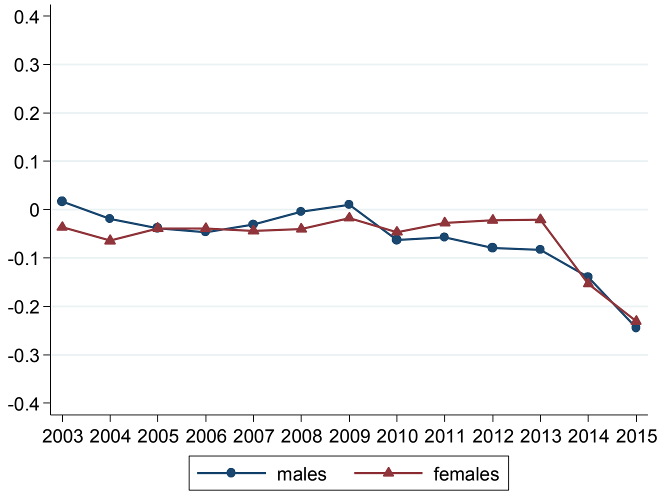


Figure 6.2.

Correlation coefficient for those born 1938–1948 and at least 30 years of contribution, Sweden

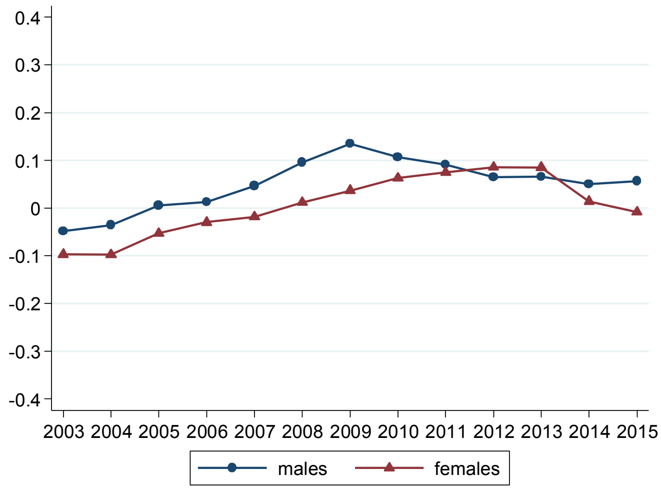
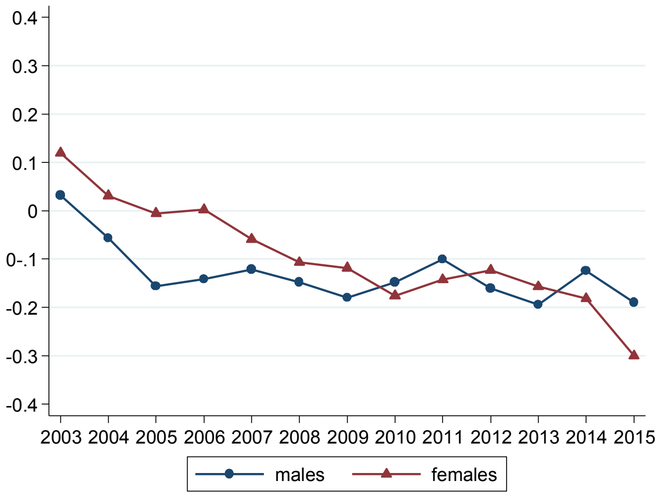


Figure 6.2 only includes retirees with 30 or more contributing years. There is almost no correlation between the retirement age and the contribution length. There is a small increase in the correlation coefficient over time. This can be explained by the fact that there is only earnings data from 1960 and that the first cohorts can have years of contribution that we are unaware of.

For those with fewer than 30 years of contribution, there is a negative correlation that is decreasing over time between pension age and contributive years (Figure 6.3). This implies that there is a tendency that those with few years of contribution have higher retirement age than those with more contribution years. Since the guaranteed pension is only paid from 65 years of age, persons with few contribution years wait until 65 before retiring. The old DB system was also more lax concerning lifetime income. The 15 highest income years out of 30 were used for the pension level calculation; the phasing-out this rule might partly explain the decrease of correlation over time.

Figure 6.3.

Correlation coefficient for those born 1938–1948 and with less than 30 years of contribution, Sweden



The correlation coefficient between retirement age and the number of contribution years in Sweden has been around zero and its absolute value has been weaker than the Austria and Hungary. This might be caused by the higher minimum retirement age and the emergence of a defined contribution system with no minimum years for qualification. It is also quite difficult to draw any general conclusions since the employees studied are in a transition from a defined benefit system to a defined contribution system.

7. CONCLUSIONS

We have compared the mandatory (mainly public) pension systems of four EU countries: Hungary, Austria, Germany and Sweden from a single point of view: what was the value of the correlation coefficient between retirement age and length of contribution between 2000–2015 or in a long subinterval? In each country, the public pension pillar has been dominating, though in Sweden the private pillar has also been playing an important role (OECD, 2016). The correlation between the retirement age and the employment length varied from year to year and country to country. Austria (still counting only a limited number of years' earning to define the lifetime reference wage, lower female than male normal retirement age and with a weak deduction for early retirement) has the most generous system in offering early retirement for workers with short and fragmented careers. On the other end, Germany and Sweden with their point system and nonfinancial defined contribution system respectively are close to copy a mandatory life-annuity scheme. Hungary (the only ex-communist country, still suffering from the transitional depression) had improved the incentives to delay retirement until 2010, but since 2011, politically motivated rules (no deduction for females with 40 years of employment and no early retirement for others) crept in.

Gender differences appear predominantly in Hungary and Austria where males and females have had different opportunities for earlier retirement. In Germany, these differences are diminishing over time because the access to the specific early retirement scheme for females has been impeded. In Sweden there seem to be no consistent gender difference because there are not any specific early retirement regulations for females.

Rather than studying the social optimality of these pension systems, we have used a very simple and sturdy indicator for evaluation: the correlation coefficient between the retirement age and the length of employment. If there were no fragmentation or the fragmentation were quasi homogenous, then the correlation coefficient would be close to +1. With heterogeneous workers and unnaturally favoring the early retirement of those with long contribution lengths, the correlation approaches –1. As was expected, positive correlation has appeared in all the three mature market economies, but surprisingly, their values remained close to 0.

APPENDIX.

THE IMPACT OF THE RETIREMENT RULES ON THE CORRELATION

In this Appendix two simple models are constructed, where the impact of the retirement rules on the correlation coefficient between the retirement age and the contribution length can be studied theoretically. Model 1 (rigid) produces a negative correlation, while Model 2 (flexible) produces a positive correlation. In fact, rather than calculating correlation coefficients, we rely on Csebyshev's algebraic inequality (cf. Simonovits, 1995 and Wagener, 1996) and prove that because the retirement age is an increasing (decreasing) function of the degree of fragmentation in Model 1 (2), while the contribution length is a decreasing function of it.

Our starting point is that there are groups in the population whose working carriers are differently fragmented. We do not repeat the formulas from Section 2 but change their content: now the index i refers to a group rather than an individual, n is the number of groups rather than that of persons, therefore f_i (> 0) be the population share of group i with $\sum_i f_i = 1$. Correspondingly,

$$\mathbf{E}R = \sum_i f_i R_i \quad \text{and} \quad \mathbf{E}S = \sum_i f_i S_i \, .$$

MODEL 1 (RIGID)

The simplest way to model a rigid retirement rule is the following. For every worker, the retirement period's length $D - R$ should be equal to a given share σ (<1) to the contribution length $S = \varphi(R - L)$, implying $D - R = \sigma\, \varphi(R - L)$. Hence the φ-specific retirement age and the corresponding contribution length are respectively equal to

$$R(\phi) = \frac{(\sigma\phi L + D)}{\sigma\phi + 1} \quad \text{and} \quad S(\phi) = \frac{\phi(D - L)}{\sigma\phi + 1}$$

Here (R(φ), S(φ)) depend on the non-fragmentation rate φ; the retirement age is decreasing in φ and the contribution length is increasing in φ. Obviously, this implies a negative correlation. Defining the gross replacement rate (β) by $b = \beta\, w$, the equality of lifetime contributions and benefits, $\tau\, \varphi(R - L)\, w = b\,(D-R)$ implies $\beta = \tau/\sigma$.

Table A.1 displays these functions for three values of φ, setting σ=1/2, L=20, D = 80. As is expected, the stronger the fragmentation, the higher the retirement age and the shorter is the contribution length.

Table A.1

Fragmentation, retirement age and contribution length

Non-fragmentation rate	Retirement age (year)	Contribution length (year)
1.0	60.0	40.0
0.9	61.4	37.2
0.8	62.9	34.3

Of course, this rule––insisting on equalizing the replacement rates for all workers––is very rude but captures the thinking of those planners who punish workers of short employment with late retirement. Due to the quasi-linearity of $R(\varphi)$ and $S(\varphi)$, their correlation is very close –1.

MODEL 2 (FLEXIBLE)

In contrast, Model 2 is based on a standard optimization. Here our initial idea can be proved: in a NDC system, *the more fragmented one's labor career, the earlier she retires*. The simplest formulation of NDC' benefit is obtained by dividing lifetime contributions $\tau\ \varphi\ (R–L)$ into $D–R$ equal parts:

$$b(R) = \frac{\tau\phi(R - L)}{D - R}$$

Let us assume that our worker has a log-linear utility function: annual consumption yields utility $\log(1 - \tau)$ and labor disutility is ξ, i.e. her annual utility while working is equal to $\log(1-\tau) - \xi$. We assume that when the person is out work but not yet retired, she receives a modest social aid a and has the same disutility ξ as a worker has (moral disutility is equal to physical disutility). Then the discounted lifetime utility function (with discount factor δ) is given by

$$U[R] = \varphi\ (R - L)\ [\log\ (1 - \tau) - \xi] + (1 - \varphi)(R - L)\ [\log a - \xi] + \delta(D - R)\log b(R).$$

Inserting the NDC-formula into the utility function, yields

$$U(R) = \varphi\ (R - L)\ [\log\ (1 - \tau) - \xi] + (1 - \varphi)(R - L)\ [\log a - \xi]$$
$$+ \delta\ (D - R)[\log \tau + \log \varphi + \log\ (R - L) - \log\ (D - R)].$$

The optimal retirement age is determined by the first-order stationarity condition:

$$0 = U'(R) = \varphi\,[\log(1-\tau) - \xi] + (1-\varphi)\,[\log a - \xi]$$

$$- \delta[\log \tau + \log \varphi + \log(R-L) - \log(D-R)] + \frac{\delta(D-R)}{R-L}.$$

There is no explicit solution $R(\varphi)$ to this equation but the character of the dependence of the optimal retirement age on the non-fragmentation rate can be determined. Indeed, denoting $U'(R)$ by $V(\varphi, R)$, we shall rely on the theorem on the implicit function: $R'(\varphi) = -V'_{\varphi}(\varphi, R)/V'_{R}(\varphi, R)$. Use the second-order condition $V'_{R}(\varphi, R) < 0$ and $V'_{\varphi}(\varphi, R) = \log(1-\tau) - \xi - \log a + \xi - \delta/\varphi$. Then $R'(\varphi) > 0$ if and only if $0 < V'_{\varphi}(\varphi, R)$. Since $a < 1-\tau$, $\varphi > \delta/[\log(1-\tau) - \log a]$ must hold. To have a nonempty interval for φ, even if $\delta = 1$, $a < (1-\tau)\,e$ is to hold, where e ≈2,718... is the base of natural logarithm.

For example, for $\tau = 0.2$, $a = 0.2$ and $\delta = 1$, we have an increasing function $R(\varphi)$ in the interval (0.72, 1). According to Table A.2, as the fragmentation $1-\varphi$ increases from 0 to 0.2, the optimal retirement age drops from 62.9 to 62.1 year, and the corresponding contribution length also drops from 42.6 to 33.7 years.

Table A.2

Fragmentation, optimal retirement age and contribution length

Non-fragmentation rate	Optimal retirement age (year)	Contribution length (year)
1.0	62.9	42.6
0.9	62.3	38.1
0.8	62.1	33.7

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