Riskiness, Risk Aversion, and Risk Sharing: The Comparative Statics of Informal Insurance*

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September 2008

Abstract

This paper examines how informal insurance, characterized by limited commitment, depends on risk preferences and the riskiness of income. The level of informal insurance is defined as the reciprocal of the discount factor above which perfect risk sharing is self-enforcing. When complete insurance is possible, that is, there is no background risk, there is more risk sharing, if (i) the utility function is more concave, and if (ii) income is more risky considering a mean-preserving spread, or an SSD deterioration. However, (ii) no longer holds when insurance can only be incomplete, because of the interplay of idiosyncratic and aggregate risk. In the case of CARA (CRRA) preferences, informal insurance depends positively on both the coefficient of absolute (relative) risk aversion and the standard deviation (coefficient of variation) of the distribution that generates income, and is independent of mean income.

Keywords: informal insurance, limited commitment, risk preferences, comparative statics, dynamic stochastic games, measuring risk

JEL codes: D11, D80

*Preliminary, comments welcome. I thank Pierre Dubois, Johannes Gierlinger, Christian Gollier, Bruno Jullien, Gergely Lakos, Thierry Magnac, Markus Reisinger, Karl Schlag, and participants of the Risk Sharing workshop in Toulouse, the Summer workshop in Budapest at the Economic Institute - Hungarian Academy of Sciences, and ESEM in Milan for useful comments and suggestions. All remaining errors are mine.

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1 Introduction

Informal insurance occurs in a wide variety of economic contexts. Two neighbors in a village will help each other out, when one faces some negative shock, like illness, or crop loss due to pests. Members of a family also insure one another informally by, for example, helping out a member who becomes unemployed. Governments help one another in case of a natural disaster, or a currency crisis. An employer and her employee insure each other against the fluctuations of the market wage. However, in all these cases, transfers are not enforceable by law, one party could walk away from the informal insurance arrangement at any time.

Risk theorists have devoted a lot of attention to formal insurance contracts, that occur between a risk-averse agent and an insurance company. The first issue is to measure the level of insurance, that is, "how much?" insurance occurs. In the case of formal insurance, the answer is simple: we can measure insurance in money units. The second issue is to relate insurance to risk preferences and the riskiness of a random variable, or gamble. With appropriate measures of risk aversion and riskiness, we would like to have comparative static results like "if the agent is more risk averse, she is willing to pay more to avoid a given gamble", and "a risk-averse agent is willing to pay more to avoid a riskier gamble".

This paper aims to follow the same route considering informal insurance. Thus, first, a way to measure the level of informal insurance is proposed. Then, this paper examines how risk preferences and riskiness of agents' income together determine voluntary insurance, and aims to establish the type of comparative static results that exist for the case of formal insurance.

To do this, two agents are considered, who are allowed to share risk informally. The second agent replaces the principal, the insurance company, of formal contracting to serve as a risk sharing partner. There is no formal, enforceable insurance contract available, and transfers between agents have to be self-enforcing. The level of informal insurance achieved will be naturally influenced by agents' risk preferences and the riskiness of the random income they face. To my knowledge, this paper is the first to examine the comparative statics for the relation between riskiness, risk aversion, and *informal* insurance in a systematic way.

Informal insurance is modeled by risk sharing with limited commitment in a dynamic context, following Thomas and Worrall (1988), Kocherlakota (1996), Ligon, Thomas, and Worrall (2002), among others. To fix ideas, suppose that the economy is populated by two ex-ante identical, infinitely-lived, risk-averse agents. Income may take a discrete number of positive and finite values, and is distributed according to some random variable Y independently and identically (i.i.d.) across time for both agents. A realization of Y gives each agent's endowment, or income, each period, that she may consume or transfer to her risk sharing partner. Note that we are interested in insurance against the (static) random variable Y, but to talk about informal insurance, we need to embed it in a dynamic model. This is because transfers between agents have to be voluntary, or, self-enforcing. That is, in every period and state of the world, each agent may renege on the contract, and consume her own income in every subsequent period. What makes transfers possible today is the expected gain from future insurance.

The solution of the model of informal insurance is a rather complicated object. In particular, today's consumption allocation depends not only on today's income realizations, but also on last period's consumption allocation through the ratio of marginal utilities. Therefore, finding a scalar measure to describe the level of informal insurance is not evident. We know that in the dynamic model of risk sharing with limited commitment, informal insurance achieved by agents depends not only on their risk preferences and the riskiness of income they earn, but also on how they value the present relative to the future, that is, the discount factor, δ. The idea is then to get rid of δ by using if to describe the level of informal insurance.

Trivially, if an agent has a high preference for the present, that is, a low discount factor, she will be less willing to make a transfer today. On the other hand, as the discount factor approaches 1, perfect risk sharing, the first best, becomes self-enforcing, according to the well-known folk theorem result. Turning this around, one may simply characterize the level of informal insurance by the discount factor above which perfect risk sharing is achieved. Let us call this value of the discount factor δ^*. Note that a lower δ^* means that more informal insurance is achieved between the agents, thus I will examine its reciprocal, $1/\delta^*$, that I call the *level of informal insurance.* δ^* could also be interpreted as a measure of (non)cooperation in an infinitely repeated games context.

After setting up the model, section 2 shows how to find δ^*, the discount factor above which perfect risk sharing is self-enforcing. Intuitively, the level of informal insurance, $1/\delta^*$, will be determined by the trade-off between the expected future gains of insurance and the cost of making a transfer today. Section 2 also presents a numerical example to motivate measuring informal risk sharing by the scalar value $1/\delta^*$. I show how $1/\delta^*$ is related to insurance transfers and the resulting consumption allocation.

Afterwards, sections 3 to 5 look at some comparative statics related to risk aversion and riskiness. First, in section 3, I examine the two most widely-used preference classes, namely, utility functions characterized by constant absolute risk aversion (CARA), and those characterized by constant relative risk aversion (CRRA). I show that in the case of CARA (CRRA) preferences, the level of informal insurance depends positively on both the coefficient of absolute (relative) risk aversion and the standard deviation (coefficient of variation) of the income process, and it is independent of mean income. These claims are in line with standard results in the literature, but provide additional justification for using the standard deviation (coefficient of variation) as the measure of riskiness in the CARA (CRRA) case. Note also that $1/\delta^*$ measures riskiness in accordance with the standard measures in the CARA and CRRA cases, and that it disentangles the effects of risk and expected value.

Then, I turn to general, increasing and concave utility function. Two scenarios are considered. The first case, that I call the *complete insurance* case, is when perfect risk sharing results in completely smooth consumption across states and time, in other words, there is no background risk. This is equivalent to there being no aggregate uncertainty. The second case is when agents still suffer from consumption fluctuations, even though they share risk perfectly. I call this case the *incomplete insurance* case. This may be thought of as a setting where there is background risk, or the case with aggregate uncertainty.

In the complete insurance case (section 4), this paper first shows that the level of informal insurance, $1/\delta^*$, depends positively on the concavity of the utility function. That is, if agents are more risk averse, they achieve more informal insurance. Then, I examine how $1/\delta^*$ depends on riskiness. I demonstrate that, under some conditions, informal insurance increases as a result of a mean-preserving spread on Y or a deterioration in the sense of second-order stochastic dominance (SSD). Thus, with common measures of risk aversion and riskiness, the desired comparative static results hold. These results also suggest that risk aversion and riskiness are inherently interdependent.

In section 5, I turn to the incomplete insurance case. First, I show that the comparative static result, that more risk aversion leads to more informal insurance, goes through, under some restriction. Then, I establish, by way of counterexamples, that a mean-preserving spread does not always result in a higher level of informal insurance. This comes from the presence of both idiosyncratic and aggregate risk. I also show that this negative result can be reconciled. In particular, keeping aggregate risk constant, the desirable comparative static result holds.

Before concluding, this paper discusses $1/\delta^*$ as a measure of riskiness, taking risk faced by agents to be inherently related to risk preferences. Useful properties include that it disentangles risk from expected value. Further, it measures riskiness by a scalar, and distributions with different means can be compared as well. $1/\delta^*$ characterizes the riskiness of the environment as agents experience it. It allows one to compare the riskiness of any two distributions, given risk preferences. Further, given the income distribution, one is able to rank preferences in terms of how risk averse they are with respect to the risk faced.

The rest of the paper is structured as follows. First, the related literature is discussed. Then, section 2 presents a model of risk sharing with limited commitment, and shows how to determine δ^*, the discount factor above which perfect risk sharing is self-enforcing. A numerical example is also presented that relates δ^* to the solution of the model and the insurance transfers. Section 3 looks at CARA and CRRA preferences. Then general utility functions are considered. Section 4 contains the comparative static results on how the level of informal insurance $1/\delta^*$ changes with risk aversion and riskiness, when complete insurance is possible, that is, there is no background risk. Section 5 studies which results extend to the case when insurance can only be incomplete. Section 6 discusses measuring risk. Section 7 concludes.

1.1 Related literature

This paper builds on two branches of the literature. First of all, it is related to the theories of comparative risk and risk aversion. Second, it examines informal insurance arrangements. In particular, informal risk sharing is modeled by requiring insurance contracts to be self-enforcing, that is, commitment is limited. I discuss these two branches in turn.

In the literature on comparative risk and risk aversion the aim is to find appropriate measures of risk aversion and the riskiness of a random variable in order to have the "right" comparative static results. Namely, a more risk-averse agent should invest less in the more risky asset, and should be willing to pay more for formal insurance to avoid a given gamble. And, any risk-averse agent should invest less in the risky asset when its riskiness increases, and should be willing to pay more to avoid a riskier gamble.

The first issue is measuring risk aversion. The classic measure is due to Arrow (1965) and Pratt (1964). If agent i is more risk averse in the sense of Arrow-Pratt than agent j, then i's utility function is an increasing and concave transformation of that of j. The Arrow-Pratt coefficient of absolute risk aversion is widely used, including this paper, and it has some useful properties. Namely, if agent i is more risk averse than agent j, then (i) i will invest less in the risky asset in a simple two-asset portfolio model, and (ii) i will be willing to pay more for insurance against having to face some given gamble. However, if both assets are risky, or only incomplete insurance is available against a gamble, in other words, there is background risk, the Arrow-Pratt measure no longer has nice properties. (Gollier, 2001)

Ross (1981) defines a somewhat stronger measure of risk aversion, that has the desirable comparative static properties. Jewitt (1987) finds conditions for the comparative static results (i) and (ii) in the presence of background risk, keeping the Arrow-Pratt measure of risk aversion, but putting stronger restrictions on the stochastic processes.

Turning to measuring “variability”, or risk, of a random process, the most well-known measure is probably second-order stochastic dominance (SSD), due to Hadar and Russell (1969), and Rothschild and Stiglitz (1970). An SSD change in the distribution can be deconstructed into a sequence of mean-preserving spreads, plus maybe a decrease in the mean. However, SSD is a partial order, and presents a strong requirement. Note that the dominated process cannot have a higher mean. Jewitt (1989) develops a way, based on SSD, to measure the riskiness of processes with different means. He derives comparative static results related to the concavity of the utility function and the choice between two risky assets. This paper aims to establish similar results with respect to informal insurance.

The literature on informal insurance has been developed to explain the partial insurance observed empirically. The idea is that two agents may enter into a risk sharing arrangement to mitigate the adverse effects of risk they face, even when formal insurance contracts are not available. A natural assumption is to require these informal contracts to be self-enforcing. The model has a wide range of interpretations. One may have in mind households in a village (Ligon, Thomas, and Worrall, 2002), members of a family (Mazzocco, 2007), an employee and an employer (Thomas and Worrall, 1988), or countries (Kehoe and Perri, 2002). Further, Schechter (2007) uses the same model to examine the interaction between a farmer and a thief, and Dixit, Grossman, and Gul (2000) use a similar model to examine cooperation between opposing political parties[1].

Kimball (1988) was the first to argue that informal risk sharing in a community may be achieved with voluntary participation of all members. His computations for the constant relative risk aversion (CRRA) case also suggest that risk sharing arrangements are less likely to exist, the lower the discount factor. Thomas and Worrall (1988) build a model of two-sided limited commitment in a dynamic wage contract setting. Early contributions to modeling risk sharing with limited commitment include Coate and Ravallion (1993), who introduce two-sided limited commitment in a dynamic model, but they restrict contracts to be static. They compute transfers assuming CRRA preferences as a function of the discount factor and the coefficient of relative risk aversion. However, their characterization of transfers is not

[1] I thank Refik Emre Aytimur for this reference.

optimal, once we allow for history-dependent contracts. Fafchamps and Lund (2003) argue that enforcement constraints play an important role in informal insurance arrangements, based on evidence from rural Philippines.

Kocherlakota (1996) allows for dynamic contracts, and proves existence and some important properties of the solution, but he does not give an explicit characterization. Ligon, Thomas, and Worrall (2002) characterize and calculate the solution of a dynamic model of risk sharing with limited commitment. They also look at the effect of changing the discount factor. In particular, they prove that there exists a discount factor, δ^*, above which perfect risk sharing is self-enforcing, and there also exists a discount factor, δ^{**}, below which agents stay in autarky. Genicot and Ray (2002) give a sufficient condition for nontrivial risk sharing contracts to exist, that is, they characterize δ^{**}. This paper deals with δ^*.

Some papers in the literature consider particular issues related to the comparative statics of informal insurance. Genicot (2006) examines how the likelihood of perfect risk sharing changes with wealth inequality, in the case where preferences are characterized by hyperbolic absolute risk aversion (HARA). Dubois (2006) considers quadratic utility, and shows that the value of perfect risk sharing relative to autarky is increasing in risk aversion. Krueger and Perri (2006) argue that more cross-sectional income inequality leads to more insurance, thus cross-sectional consumption inequality increases less, or may even decrease. More cross-sectional income inequality is in fact equivalent to more volatile income. Fafchamps (1999) shows that in the case of a static contract, under some conditions, one can always find a concave transformation of the utility function, or a mean-preserving spread, that destroys the sustainability of the risk sharing arrangement (see Fafchamps (1999), Proposition 3). This paper establishes conditions under which the desirable comparative static results hold for the prefect risk sharing contract to be sustainable, or, self-enforcing.

2 The level of informal insurance

This section first sets up the model of informal insurance. In particular, I use a model of risk sharing with limited commitment, following Thomas and Worrall (1988), Kocherlakota

(1996), Ligon, Thomas, and Worrall (2002), and others. Then, section 2.2 looks at a numerical example, and relates the solution of the model to δ^*, the discount factor above which perfect risk sharing is self-enforcing. Afterwards, section 2.3 shows how to find δ^*. The *level of informal insurance* is then defined as its reciprocal, $1/\delta^*$.

2.1 Modeling informal insurance

Consider an economy with two infinitely-lived, risk-averse agents[2], who receive a stochastic endowment, or income, each period. Note that, in this paper, income is the sum of any exogenous revenue, plus the payoff from any gamble "played". Note further that risk is exogenous, agents cannot choose not to play the gamble. Suppose that income follows a discrete distribution, Y, with positive and finite possible realizations, and is independently and identically distributed (i.i.d.) across time periods[3]. To attenuate the adverse effects of the risk they face, agents may enter into an informal risk sharing arrangement. The informal risk sharing contract has to be self-enforcing, that is, at each state of the world and time period, each agent has to be at least as well off respecting the terms of the contract, as consuming her own income today and in all subsequent periods. Consumption smoothing across states of nature, and not across time is considered here, and I assume that no savings, or storage is possible. Further, agents hold the same beliefs about the income processes ex ante, and income realizations are common knowledge ex post.

To fix ideas, denote by s the state of the world realized, and $y_i(s)$ the income realization for agent i at state s. Denote the utility function $u()$, defined over a single, private, and perishable consumption good c. Suppose that $u()$ is strictly increasing, twice continuously differentiable, strictly concave, so agents are risk averse, and egoïstic in the sense that agents only care about their own consumption. Each agent $i \in \{1, 2\}$ maximizes her expected lifetime utility,

$$E_0 \sum_t \delta^t u\left(c_{it}(s)\right), \tag{1}$$

[2] The model can easily be extended to n agents.

[3] The model can be extended to the case where income follows a Markov-chain. For the purposes of the present paper, however, the i.i.d. assumption is not restrictive, as argued above.

where E_0 is the expected value at time 0 calculated with respect to the probability measure describing the common beliefs, $\delta \in (0,1)$ is the (common) discount factor, and $c_{it}(s)$ is consumption of agent i at state s and time t.

In terms of how much informal insurance is achieved, three cases are possible. Given the utility functions, the discount factor, and the income processes, at the constrained-efficient solution there might be (i) no risk sharing, that is, agents stay in autarky, (ii) perfect risk sharing, or (iii) something in between, that is, partial insurance. Let us briefly look at each case in turn.

First of all, in autarky the agents' maximization problem is trivial, since resources are not transferable across time. Each agent consumes her own income in each state of the world and time period. The expected lifetime utility of agent i, at period t and state $\widetilde{s}$, can be written as

$$u\left(y_i\left(\widetilde{s}\right)\right) + \frac{\delta}{1-\delta}\sum_s Pr\left(s\right) u\left(y_i\left(s\right)\right), \tag{2}$$

where $Pr\left(s\right)$ is the probability of state s occurring. Note that, by definition, the informal risk sharing contact must provide at least the lifetime utility (2), in each state $\widetilde{s}$ and at each time t, for agents to voluntary participate. That is, the enforcement constraints, or, ex-post[4] participation constraints, must be satisfied.

Second, in the case of perfect risk sharing, all idiosyncratic risk is eliminated. To find the perfect risk sharing solution, or, the set of Pareto-optimal allocations, one may consider the social planner's problem. In particular, the (utilitarian) social planner maximizes a weighted sum of agents' lifetime utilities. In mathematical terms, the social planner's objective is

$$\sum_t \sum_s \delta^t Pr\left(s\right) u\left(c_{1t}\left(s\right)\right) + x_0 \sum_t \sum_s \delta^t Pr\left(s\right) u\left(c_{2t}\left(s\right)\right), \tag{3}$$

where x_0 is the (initial) relative weight of agent 2 in the social planner's objective. The social planner maximizes (3) such that the resource constraint,

$$c_{1t}\left(s\right) + c_{2t}\left(s\right) = y_{1t}\left(s\right) + y_{2t}\left(s\right), \tag{4}$$

is satisfied, for all t and s.

[4]"Ex post" here means after the realization of the state of the world, at each time period.

The first order conditions yield the standard result that

$$\frac{u'\left(c_{1t}\left(s\right)\right)}{u'\left(c_{2t}\left(s\right)\right)} = x_0, \forall s, \forall t, \tag{5}$$

that is, the ratio of marginal utilities is constant across time and states of nature in the case of perfect risk sharing. Replacing for $c_{2t}\left(s\right)$ in (5) using the resource constraint (4), the consumption allocation can be easily solved for. Let $c_1^*\left(s, x_0\right)$ and $c_2^*\left(s, x_0\right)$ denote the solution, in other words, the sharing rule. The consumption allocation does not depend on the time period t, since savings are assumed away. The expected lifetime utility for agent i at state $\widetilde{s}$, in the case of perfect risk sharing, can be written as

$$u\left(c_i^*\left(\widetilde{s}, x_0\right)\right) + \frac{\delta}{1-\delta}\sum_s Pr\left(s\right) u\left(c_i^*\left(s, x_0\right)\right). \tag{6}$$

Note that the consumption allocation only depends on aggregate income[5] and the relative weight of agent 2 in the social planner's objective.[6] Thus we may also write the sharing rule as $c_1^*\left(y_1\left(s\right)+y_2\left(s\right), x_0\right)$ and $c_2^*\left(y_1\left(s\right)+y_2\left(s\right), x_0\right)$.

The third case is when some, but not perfect insurance is achieved. This case is often referred to as partial insurance. Here the perfect risk sharing solution is not self-enforcing, there is at least one enforcement constraint that binds. This means that at some state $\widetilde{s}$ for one of the agents, the lifetime utility from perfect risk sharing (6) would be smaller than the autarky utility (2). In such states the informal risk sharing contract will determine an allocation such that this agent is indifferent between respecting the terms of the contract or deviating to autarky. The constrained-efficient contract can be found using numerical dynamic programming (see Laczó (2008) for an algorithm).

The solution in the limited commitment case can be fully characterized by a set of state-dependent intervals on the relative weight of household 2, or, the ratio of marginal utilities, x, that give the possible relative weights in each income state. Denote the interval for state s by $[\underline{x}^s, \overline{x}^s]$. Suppose that last period the ratio of marginal utilities was x_{t-1}, and today

[5]This property is sometimes referred to as the mutuality principle.

[6]Note that in addition to (4) and (5), ex-ante participation constraints have to be satisfied. These constraints restrict the set of x_0 for which the agents are willing to "sign" the risk sharing contract at time 0. See Laczó (2008).

the income state is s. Today's ratio of marginal utilities, x_t, is determined by the following updating rule (Ligon, Thomas, and Worrall, 2002):

$$x_t = \begin{cases} \overline{x}^s & \text{if} \quad x_{t-1} > \overline{x}^s \\ x_{t-1} & \text{if} \quad x_{t-1} \in [\underline{x}^s, \overline{x}^s] \\ \underline{x}^s & \text{if} \quad x_{t-1} < \underline{x}^s \end{cases}$$

When an enforcement constraint binds, we cannot keep x constant (as in the perfect risk sharing case). However, intuitively, we will try to keep x_t as close as possible to x_{t-1}. The constrained-efficient solution has a number of interesting properties, including history dependence, and a quasi-credit element (Fafchamps, 1999). Details are given in Kocherlakota (1996), Ligon, Thomas, and Worrall (2002), and Laczó (2008), among others. Note that, in the case when perfect risk sharing is self-enforcing, all the $[\underline{x}^s, \overline{x}^s]$ overlap, that is, there exists some $\tilde{x}$ such that $\tilde{x} \in [\underline{x}^s, \overline{x}^s]$, $\forall s$. Further, if such an $\tilde{x}$ exists, then it will be reached with probability 1 after a sufficient number of periods (Kocherlakota, 1996).

Let us examine how the above three cases evolve as the discount factor, δ, changes. Take risk preferences and the income processes given. For δ approaching zero, the agent receiving high income today will not make a transfer, since she values current consumption too much. Thus, for low values of δ, we are in the autarky case. On the other extreme, when δ is close to 1, future consumption is almost as important as present consumption, and average utility across time will be maximized, which can be achieved sharing risk perfectly. This is the well-known folk theorem result that the first best is achieved for a sufficiently high discount factor. Finally, for some intermediate values of δ, partial insurance occurs, that is, we are in the third case. Further, there exists a level of the discount factor, given preferences and the income process, above which perfect risk sharing is self-enforcing (Ligon, Thomas, and Worrall (2002), Proposition 2, part (ii)). Denote this discount factor by δ^*.

2.2 A numerical example

Let us reconsider the numerical example presented in Ligon, Thomas, and Worrall (2002). Suppose that there are two agents with isoelastic utility with a coefficient of relative risk aversion equal to 1, that is, $u() = ln()$. Income is independently and identically distributed (i.i.d.) across agents and time, and it may take two values, high ($y^h = 20$, say) or low (y^l=10)[7]. The probability of the low income realization is 0.1. Note that there are four income states: two symmetric ones, hh (both agents earn y^h) and ll (both get y^l), and two asymmetric ones hl (agent 1 gets y^h and agent 2 y^l) and lh (the reverse). Note that in the perfect risk sharing case with equal weight, aggregate income should always be shared equally. This means that in the asymmetric states a transfer of 5 should be made, thus both agents consume 15.

Let us also consider an alternative scenario where the income distribution is as before, but agents are more risk averse. Denote the new utility function by $v()$. Let the coefficient of relative risk aversion be constant and equal to 1.5, thus $v(c) = c^{1-\sigma}/(1-\sigma) = c^{-0.5}/-0.5$.

The aim of this exercise is to compare the solution of the risk sharing with limited commitment model in these two cases. In particular, we first look at the optimal state-dependent intervals on the ratio of marginal utilities, that fully characterize the solution, as a function of the discount factor. I consider discount factors between 0.84 and 0.99. Then I examine what δ^*, the discount factor above which perfect risk sharing is self enforcing, tells us about the solution, and how it is related to the insurance transfers and the consumption allocation. The computations have been done using the software R (see www.r-project.org).

The black lines in Figure 1 reproduce Figure 1 in Ligon, Thomas, and Worrall (2002), that represents the logarithm of x, the ratio of marginal utilities, as a function of the discount factor, δ. The dashed lines represent the optimal intervals for states hh and ll (the two coincide with logarithmic utility), while the solid lines are the intervals for the asymmetric states. The blue lines in Figure 1 show the corresponding intervals when $\sigma = 1.5$, that is, when agents are more risk averse.

First of all, let us look at the case where $\delta = 0.94$. For this discount factor, all the intervals overlap, except for the ones for states hl and lh (see the intervals along the vertical, dotted line in Figure 1). Then, the ratio of marginal utilities, after a sufficient number of periods, will only take two values, $\overline{x}^{hl}$ and $\underline{x}^{lh} = 1/\overline{x}^{hl}$. For the utility function $u()$, these numbers

[7] The graph is the same for any y^h and y^l, if $y^l = 0.5y^h$ holds. The transfers and consumptions will be different, of course.

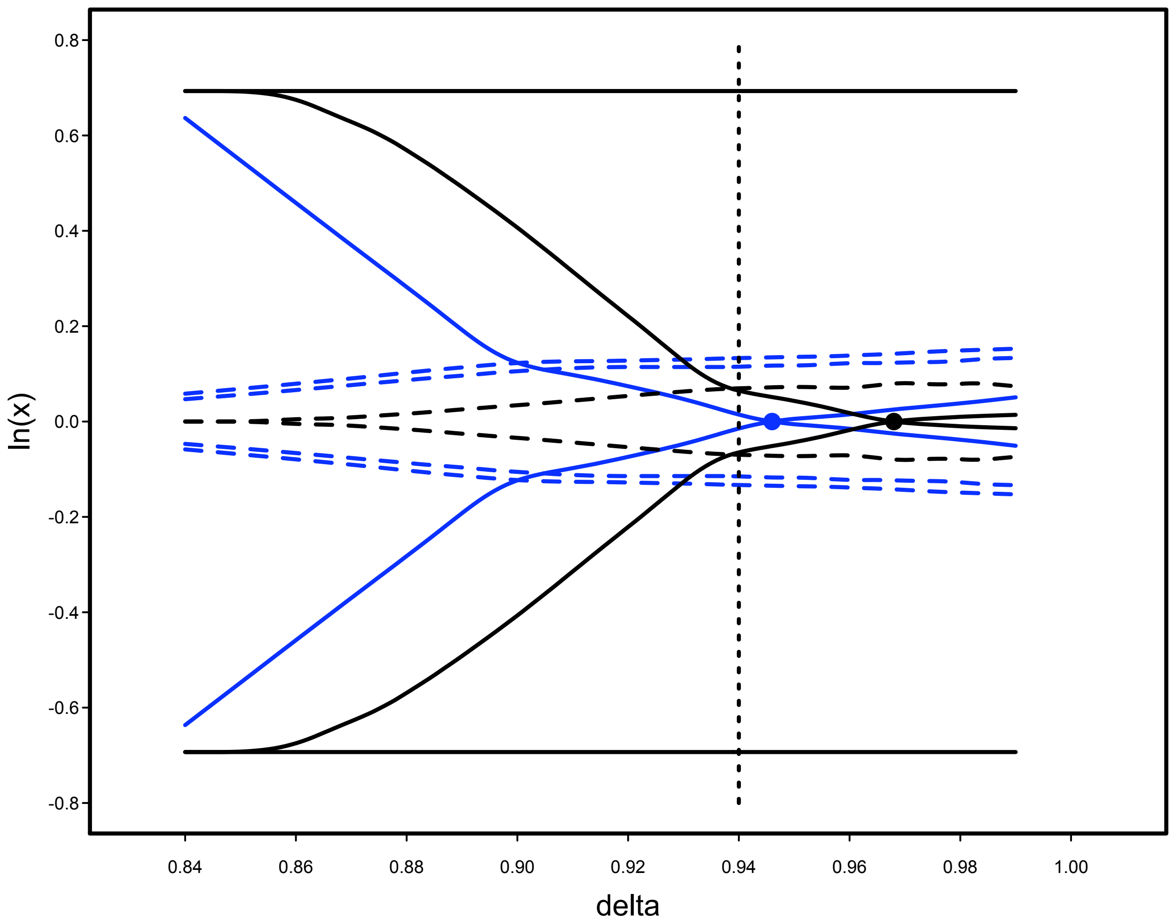

Figure 1: The optimal intervals of $ln\,(x)$ as a function of δ. The black lines show the optimal intervals on the (logarithm of the) ratio marginal utilities for the utility function $ln(c)$ (as in Ligon, Thomas, and Worrall (2002)), and the blue lines for $c^{1-\sigma}/(1-\sigma)$ with $\sigma = 1.5$. The dots represent δ^{*}. See more details in the main text.

are 0.940 any 1.064. When agents' preferences are described by the more concave function $v()$, they equal 0.990 and 1.010. It follows from the first order conditions that the insurance transfers in the asymmetric states, hl and lh, are 4.53 and 4.92, for the utility functions $u()$ and $v()$, respectively. This also means that, if agents are more risk averse, consumption is smoother across states, so agents achieve more insurance. Note also that we are very close to the first-best transfer, 5, in both cases.

Now, notice that for any discount factor, the blue intervals, that belong to the case when agents are more risk averse, are wider. This means that a wider range of x's are possible with voluntary participation. Remember that, in the case where perfect risk sharing is self-enforcing, all the intervals overlap. On the other hand, when no informal insurance is possible, that is, when agents stay in autarky, each interval is just one point. Thus, if the intervals are wider, we may say that there is more insurance.

Finally, in Figure 1, the dots represent the discount factor above which perfect risk sharing is self-enforcing, δ^*. The black dot represents $\delta^* = 0.964$ for the utility function $u()$, while the blue dot is $\delta^* = 0.943$ that belongs to the more concave utility function $v()$. Notice that, as the dot moves to the left, the optimal intervals also move to the left, thus they become wider. Thus, one may capture the changes in the intervals, and thereby the changes in the transfers and the consumption allocation, by the single point δ^*. The next section shows how to find it in general.

2.3 Determining the level of informal insurance

Now I show how to determine δ^*, the discount factor such that, for all $\delta \geq \delta^*$, perfect risk sharing is self-enforcing. In other words, we are are looking for the lowest possible discount factor such that (i) perfect risk sharing occurs, that is, the ratio of marginal utilities is constant across states and over time, denoted x^*, and (ii) the enforcement constraints are satisfied. In mathematical terms, there exists x^* such that (6), with $x^* = x_0$, is greater than (2), for all $\widetilde{s}$.

Intuitively, an enforcement constraint is most likely to bind when one agent has the highest possible income realization, y^h, while the other agent has the lowest possible one, y^l. This is when the autarky lifetime utility is highest, and when the biggest transfer should be made to respect the terms of the perfect risk sharing contract.

The expected lifetime utility of agent 1 in autarky, when her income realization is y^h today, is

$$u\left(y^h\right) + \frac{\delta}{1-\delta}\sum_s Pr\left(s\right) u\left(y_1\left(s\right)\right), \tag{7}$$

while for agent 2 it is

$$u\left(y^h\right) + \frac{\delta}{1-\delta}\sum_s Pr\left(s\right) u\left(y_2\left(s\right)\right). \tag{8}$$

Since agents are assumed ex-ante identical, that is, they have the same preferences and their income is generated from the same random variable Y, (7) and (8) are equal, and can be written as

$$u\left(y^h\right) + \frac{\delta}{1-\delta}\sum_s Pr\left(s\right) u\left(y_i\left(s\right)\right) \equiv u\left(y^h\right) + \frac{\delta}{1-\delta}Eu(y), \tag{9}$$

where $Eu(y)$ is the expected per-period utility in autarky.

The expected lifetime utility of agent 1 in the perfect risk sharing case, when she is earning y^h and agent 2 is getting y^l today, is

$$u\left(c_1^*\left(y^h + y^l, x^*\right)\right) + \frac{\delta}{1-\delta}\sum_s Pr\left(s\right) u\left(c_1^*\left(y_1\left(s\right) + y_2\left(s\right), x^*\right)\right).$$

This expression is the same as (6) with $x^* = x_0$, and making explicit that the consumption allocation depends on state s only through aggregate income. Similarly, the value of perfect risk sharing for agent 2, when her income is y^h and agent 1 is earning y^l, is

$$u\left(c_2^*\left(y^h + y^l, x^*\right)\right) + \frac{\delta}{1-\delta}\sum_s Pr\left(s\right) u\left(c_2^*\left(y_1\left(s\right) + y_2\left(s\right), x^*\right)\right).$$

One can find $c_1^*\left(y_1\left(s\right) + y_2\left(s\right), x^*\right)$ and $c_2^*\left(y_1\left(s\right) + y_2\left(s\right), x^*\right)$ using the first order conditions, equation (5).

I am looking for the lowest possible discount factor such that the following two enforcement constraints are satisfied:

$$u\left(y^h\right) + \frac{\delta}{1-\delta}Eu(y) \leq u\left(c_1^*\left(y^h + y^l, x^*\right)\right) + \frac{\delta}{1-\delta}\sum_s Pr\left(s\right) u\left(c_1^*\left(y_1\left(s\right) + y_2\left(s\right), x^*\right)\right) \tag{10}$$

and

$$u\left(y^h\right)+\frac{\delta}{1-\delta}Eu(y)\leq u\left(c_2^*\left(y^h+y^l,x^*\right)\right)+\frac{\delta}{1-\delta}\sum_s Pr\left(s\right)u\left(c_2^*\left(y_1\left(s\right)+y_2\left(s\right),x^*\right)\right), \quad (11)$$

where x^* is the ratio of marginal utilities, or, the relative weight of agent 2 in the social planner objective, that is reached after a sufficient number of periods with probability 1, starting from any initial relative weight x_0 (see Kocherlakota (1996)). Remember that, if the enforcement constraints (10) and (11), relating to the most unequal states hl and lh, respectively, are satisfied, then the enforcement constraints of all other states will be satisfied as well. Using the following lemma, finding δ^* will be easy.

Lemma 1. $x^*=1$.

Proof. Let us first distinguish three cases concerning the consumption allocation according to the value of x^*.

- $x^*=1$. Then, from (5), $u'\left(c_1^*\left(y_1\left(s\right)+y_2\left(s\right),x^*\right)\right)=u'\left(c_2^*\left(y_1\left(s\right)+y_2\left(s\right),x^*\right)\right)$. It follows immediately that $c_1^*\left(y_1\left(s\right)+y_2\left(s\right),x^*\right)=c_2^*\left(y_1\left(s\right)+y_2\left(s\right),x^*\right)=\frac{y_1(s)+y_2(s)}{2}$, $\forall s$.

- $x^*>1$. Then $u'\left(c_1^*\left(y_1\left(s\right)+y_2\left(s\right),x^*\right)\right)>u'\left(c_2^*\left(y_1\left(s\right)+y_2\left(s\right),x^*\right)\right)$, and $c_1^*\left(y_1\left(s\right)+y_2\left(s\right),x^*\right)<c_2^*\left(y_1\left(s\right)+y_2\left(s\right),x^*\right)$, since $u'()$ is decreasing. Thus in this case $c_1^*\left(y_1\left(s\right)+y_2\left(s\right),x^*\right)<\frac{y_1(s)+y_2(s)}{2}$ and $c_2^*\left(y_1\left(s\right)+y_2\left(s\right),x^*\right)>\frac{y_1(s)+y_2(s)}{2}$,$\forall s$.

- $x^*<1$. Similarly, $c_1^*\left(y_1\left(s\right)+y_2\left(s\right),x^*\right)>\frac{y_1(s)+y_2(s)}{2}$ and $c_2^*\left(y_1\left(s\right)+y_2\left(s\right),x^*\right)<\frac{y_1(s)+y_2(s)}{2}$,$\forall s$.

The proof is by contradiction. Suppose that $x^*\neq 1$, and, without loss of generality, assume further that $x^*>1$. First, note that $u\left(y^h\right)>u\left(c_i^*\left(y^h+y^l,x^*\right)\right)$ but $Eu(y)<\sum_s Pr\left(s\right)u\left(c_i^*\left(y_1\left(s\right)+y_2\left(s\right),x^*\right)\right)$, $\forall i$, thus the constraints (10) and (11) are more stringent for a lower δ. Therefore, minimizing δ, at least one of the two constraints must hold with equality. Let us consider the two cases in turn.

- (10) holds with equality. We have seen above that for $x^* > 1$, $c_1^*\left(y_1\left(s\right)+y_2\left(s\right), x^*\right) < \frac{y_1(s)+y_2(s)}{2} < c_2^*\left(y_1\left(s\right)+y_2\left(s\right), x^*\right)$, $\forall s$, thus (11) is slack. Then, (10) can be used to solve for δ^*. Rearranging (10) gives

$$\delta^* = \frac{u\left(y^h\right) - u\left(c_1^*\left(y^h + y^l, x^*\right)\right)}{u\left(y^h\right) - u\left(c_1^*\left(y^h + y^l, x^*\right)\right) + \sum_s Pr\left(s\right) u\left(c_1^*\left(y_1\left(s\right) + y_2\left(s\right), x^*\right)\right) - Eu(y)}. \quad (12)$$

  Now, consider the following alternative allocation. Transfer a small amount $\epsilon(s)$ from agent 2 to agent 1 at state s, $\forall s$, such that (11) still holds. As a result, δ^* given in (12) decreases, because the term $u\left(y^h\right) - u\left(c_1^*\left(y^h + y^l, x^*\right)\right)$ decreases, while the term $\sum_s Pr\left(s\right) u\left(c_1^*\left(y_1\left(s\right) + y_2\left(s\right), x^*\right)\right) - Eu(y)$ increases. Thus the original solution cannot be the one corresponding to the lowest δ.

- (11) holds with equality. In this case, (10) is violated, since $c_1^*\left(y_1\left(s\right)+y_2\left(s\right), x^*\right) < \frac{y_1(s)+y_2(s)}{2} < c_2^*\left(y_1\left(s\right)+y_2\left(s\right), x^*\right)$, $\forall s$.

Thus x^* cannot be different from 1, as I wanted to show. □

As a consequence, from the first order conditions (5), we know that the two agents consume the same amount, that is,

$$c_i^*(s, x^*) = c_{-i}^*(s, x^*) = \frac{y_i(s) + y_{-i}(s)}{2}, \forall s, \forall t,$$

where the index $-i$ refers to the other agent, who is not i. The expected lifetime utility of perfect risk sharing for agent i, in the state when she is getting y^h and agent $-i$ is earning y^l, can be written as

$$u\left(\frac{y^h + y^l}{2}\right) + \frac{\delta}{1-\delta} \sum_s Pr\left(s\right) u\left(\frac{y_i(s) + y_{-i}(s)}{2}\right). \quad (13)$$

Now we are ready to determine δ^* explicitly as a function of the income process and the utility function $u()$. Proposition 1 shows the formula.

Proposition 1. *The discount factor above which perfect risk sharing is self-enforcing, δ^*, is given by*

$$\delta^* = \frac{u(y^h) - u(\frac{y^h+y^l}{2})}{u(y^h) - u(\frac{y^h+y^l}{2}) + \sum_s Pr(s)(u(\frac{y_i(s)+y_{-i}(s)}{2}) - u(y_i(s)))}.$$

Proof. Equating (9) and (13), and rearranging yields the result. □

Note that a lower δ^* means that more informal insurance is achieved. Thus I define its reciprocal $1/\delta^*$ as the level of informal insurance.

Definition 1. *I call the reciprocal of the discount factor above which perfect risk sharing is self-enforcing the level of informal insurance. It is given by*

$$\frac{1}{\delta^*} = 1 + \frac{\sum_s Pr\left(s\right)\left(u\left(\frac{y_i(s)+y_{-i}(s)}{2}\right) - u\left(y_i\left(s\right)\right)\right)}{u\left(y^h\right) - u\left(\frac{y^h+y^l}{2}\right)}.$$

Since agents are ex-ante identical, if there is a state s occurring with probability $Pr\left(s\right)$, where agent i is earning $y_i\left(s\right)$ and agent $-i$ is getting $y_{-i}\left(s\right)$, then there is also a state, denoted $-s$, occurring with probability $Pr\left(-s\right) = Pr\left(s\right)$, with agent i receiving $y_i\left(-s\right) = y_{-i}\left(s\right)$ and agent $-i$ getting $y_{-i}\left(-s\right) = y_i\left(s\right)$. Therefore one may also write the level of informal insurance as

$$\frac{1}{\delta^*} = 1 + \frac{\sum_s \frac{Pr(s)}{2}\left(2u\left(\frac{y_i(s)+y_i(-s)}{2}\right) - u\left(y_i\left(s\right)\right) - u\left(y_i\left(-s\right)\right)\right)}{u\left(y^h\right) - u\left(\frac{y^h+y^l}{2}\right)}. \tag{14}$$

The second term on the right hand side is positive, because both the numerator and the denominator are positive for $u()$ increasing and strictly concave. It follows that $1/\delta^* > 1$, thus $\delta^* < 1$. δ^* is also positive, given that income realizations are bounded.

The numerator and the denominator on the right hand side of (14) have natural interpretations. The numerator is the expected future (one-period) gain of sharing risk perfectly rather than staying in autarky. The denominator is today's cost of respecting the terms of the risk sharing contract, at the state where the agent is earning y^h, while her risk sharing partner is getting y^l, that is, when respecting the contract is most costly. Using δ^* to discount future net benefits, they should be just important enough to compensate the agent for the loss she incurs today by making the transfer $y^h - \left(y^h + y^l\right)/2$.

The next two sections examine how the level of informal insurance, $1/\delta^*$ is related to risk preferences and the income distribution, Y, in particular, the riskiness of income. First, one would like to say that, if agents are more risk averse, then they achieve more informal insurance. Just as in the standard setting with a risk-averse agent purchasing insurance from an insurance company, one would like to have that a more risk-averse agent is willing to pay more to avoid a given risk. Second, similarly, if income is more risky, agents are expected to achieve more informal insurance, just as they are expected to be willing to pay more for formal insurance. First, we look at the special cases of CARA and CRRA preferences. Then, in section 4 and 5, we turn to the general case of any increasing and concave utility function.

3 CARA and CRRA preferences

This section performs comparative static exercises for the two most widely used utility functions. Namely, preferences characterized by constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) are examined in turn. I consider a simple setting with only two income realizations, y^h and y^l. Let us denote by Pr^{asym} the probability of the asymmetric states, hl and lh, occurring. Further, let $\overline{y} = \frac{y^h+y^l}{2}$ be per-capita income in the asymmetric states. In this case, the level of informal insurance simplifies to

$$\frac{1}{\delta^*} = 1 - Pr^{asym} + Pr^{asym}\frac{u\left(\overline{y}\right) - u\left(y^l\right)}{u\left(y^h\right) - u\left(\overline{y}\right)} \tag{15}$$

Extending the results to more income states is left for future work. The aim of this section is to show that, considering standard examples of parametrized utility functions, we have the desired comparative static results for $1/\delta^*$. I also investigate what the appropriate measure of riskiness is in these cases, relating riskiness to informal insurance.

3.1 CARA preferences

Suppose that the utility function $u()$ takes the standard constant-absolute-risk-aversion (CARA) form. In mathematical terms,

$$u\left(c\right) = -\frac{1}{A}exp\left(-Ac\right), \tag{16}$$

where $A > 0$ is the coefficient of absolute risk aversion. We know that, if a CARA agent, with given A and wealth, is indifferent between accepting or not accepting a gamble with given mean and standard deviation (an additive risk), then this is true for any wealth level. Further, the variance or standard deviation of a stochastic process is often used as a simple measure of risk.

Suppose that income may only take two values, and $y^h = y$ and $y^l = y - p$, with $y > 0$ and $y > p > 0$. So mean income is $y - \frac{p}{2}$, and the standard deviation is $\frac{p}{2}$. Notice that the standard deviation only depends on p and is independent of y, that I will call the level of incomes. Further, any mean - standard deviation combination can be reproduced by choosing p and y appropriately.

First of all, it is of interest to see what parameters of the model determine $1/\delta^*$, the level of informal insurance. Then, we will examine the relationship between the level of informal insurance, and (i) risk aversion, and (ii) the riskiness of income as measured by the standard deviation. In particular, I want to determine how $1/\delta^*$ changes, as (i) the coefficient of absolute risk aversion, A, increases, keeping the riskiness of the income process, the standard deviation, or p, constant, and as (ii) riskiness, p, changes, keeping the level of risk aversion, A, constant. We expect $1/\delta^*$ to increase with both A and p. This is indeed the case, as Claim 1 states.

Claim 1. *In the CARA case, $1/\delta^*$ only depends on A and p, and is independent of y. Further, $1/\delta^*$ depends positively on both A and p, that is, more informal insurance is achieved, if agents are more risk averse, or income is riskier as measured by the standard deviation.*

Proof. Replace the utility function (16), and $y^h = y$ and $y^l = y - p$ in (15), the equation determining $1/\delta^*$ for two possible income realizations. Leaving out the terms that are independent of the parameters of interest, we have

$$\frac{\left(-\frac{1}{A}\right) exp\left(-A\left(y-\frac{p}{2}\right)\right) - \left(-\frac{1}{A}\right) exp\left(-A\left(y-p\right)\right)}{\left(-\frac{1}{A}\right) exp\left(-Ay\right) - \left(-\frac{1}{A}\right) exp\left(-A\left(y-\frac{p}{2}\right)\right)},$$

which can be rewritten as

$$\frac{exp\left(Ap\right) - exp\left(\frac{1}{2}Ap\right)}{exp\left(\frac{1}{2}Ap\right) - 1} = exp\left(\frac{1}{2}Ap\right).$$

Thus $1/\delta^*$ only depends on A and p, and is independent of y, that is, of mean income. Further it is increasing in both A and p. □

Remark 1. *One may say that the correct measure of riskiness in the case of CARA preferences is the standard deviation, since, along with risk aversion, this is what determines the level of informal insurance.*

3.2 CRRA preferences

Suppose that both agents have standard constant-relative-risk-aversion (CRRA) preferences, that is

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \tag{17}$$

where $\sigma > 0$ is the coefficient of relative risk aversion, and $\sigma \neq 1$. For $\sigma = 1$, $u(c) = ln(c)$. We know that, in the case of CRRA preferences, if an agent, with given σ and wealth, is indifferent between accepting and not accepting a multiplicative risk, then this is true for any wealth level. In other words, what matters in the coefficient of variation of the gamble.

To fix ideas, suppose that $y^h = y$ and $y^l = (1-q)y$, with $y > 0$ and $0 < q < 1$. In this case, mean income is $\left(1 - \frac{q}{2}\right)y$, and the coefficient of variation is $\frac{2}{2-q}$. Notice that the coefficient of variation only depends on q, and is independent of y. Note that any mean - coefficient of variation combination can be reproduced by choosing q and y appropriately. As in the CARA case, we examine how $1/\delta^*$ depends on the parameters of the model.

Claim 2. *In the CRRA case, $1/\delta^*$ only depends on σ and q, and is independent of y. Further, $1/\delta^*$ depends positively on both σ and q, that is, more informal insurance is achieved, if agents are more risk averse, or income is riskier as measured by the coefficient of variation.*

Proof. In the appendix. □

Remark 2. *One may say that the correct measure of riskiness in the case of CRRA preferences is the coefficient of variation, since, along with risk aversion, this is what determines the level of informal insurance.*

Thus, the level of informal insurance $1/\delta^*$ is consistent with standard measures of risk aversion, and with measuring riskiness by the standard deviation (coefficient of variation), if preferences are of the CARA (CRRA) form. However, $1/\delta^*$ can be computed for any type of utility function, while it can still disentangle risk and expected value, and one can compare the riskiness of random variables with different means. See more on this in section 6.

4 The complete insurance case

Let us now turn to general preferences. This section looks at the case where, sharing risk perfectly, agents achieve *complete insurance*, that is, their consumption is completely smooth across states and over time. This is equivalent to a setting where agents in a community only face idiosyncratic risk, that is, aggregate income is the same in all states of the world. For this, the two agents' incomes must be perfectly negatively correlated. Examining informal insurance in this case is related to a standard insurance setting, where a risk-averse agent can buy complete insurance from a principal, in other words, there is no background risk.

In the complete insurance case, since aggregate income is constant across states of the world and shared equally between the two agents, consumption of both agents is equal to per-capita income, denoted $\overline{y}$. Then, (14) can be rewritten as

$$\begin{aligned}\frac{1}{\delta^*} &= 1 + \frac{\sum_s \frac{Pr(s)}{2}\left(2u\left(\overline{y}\right) - u\left(y_i\left(s\right)\right) - u\left(y_i\left(-s\right)\right)\right)}{u\left(y^h\right) - u\left(\overline{y}\right)} \\ &= 1 + \frac{u\left(\overline{y}\right) - \sum_s Pr\left(s\right) u\left(y_i\left(s\right)\right)}{u\left(y^h\right) - u\left(\overline{y}\right)} \\ &= 1 + \frac{u\left(\overline{y}\right) - u\left(CE^u\right)}{u\left(y^h\right) - u\left(\overline{y}\right)}, \end{aligned} \tag{18}$$

where CE^u is the certainty equivalent of the random income Y, when preferences are described by the utility function $u()$.

Note that the complete insurance case means putting strong restrictions on the possible income distributions. In particular, if some income $y_i\left(s\right)$ is earned with probability $Pr\left(s\right)$, then there must be another income realization $y_i\left(-s\right) = 2\overline{y} - y_i\left(s\right)$, where $2\overline{y}$ is the constant aggregate income, and it must occur with the same probability, that is, $Pr\left(-s\right) = Pr\left(s\right)$. In other words, the distribution must be symmetric.

In this section, I perform a number of comparative static exercises on how the level of informal insurance, given by equation (18), depends on the characteristics of the utility function, and the income distribution, Y. In particular, I examine how $1/\delta^*$ depends on the concavity of the utility function, that is, on risk aversion. I also study how $1/\delta^*$ changes, if the riskiness of the income distribution changes in terms of a mean-preserving spread, and when ranking the riskiness of distributions is based on second-order stochastic dominance (SSD).

First, let us compare the level of informal insurance when risk aversion changes. A standard characterization states that agent j, with utility function $v()$, is more risk averse than agent i, with utility function $u()$, if and only if $v()$ is an increasing and concave transformation of $u()$. This is equivalent to saying that agent j's (Arrow-Pratt) coefficient of absolute risk aversion is uniformly greater than that of agent i. Denote by $\phi()$ the increasing and concave function that transforms $u()$ into $v()$, that is, $v() = \phi(u())$. Taking Y as given, denote by δ_v^* (δ_u^*) the discount factor above which perfect risk sharing is self-enforcing, if agents have utility function $v()$ ($u()$).

Proposition 2. $1/\delta_v^* \geq 1/\delta_u^*$. *That is, if agents are more risk averse in the sense of having a more concave utility function, then more informal insurance is achieved.*

Proof. Using the formula determining $1/\delta^*$ with no aggregate uncertainty, equation (18), $1/\delta_v^* \geq 1/\delta_u^*$ is equivalent to

$$\frac{v\left(\overline{y}\right) - v\left(CE^v\right)}{v\left(y^h\right) - v\left(\overline{y}\right)} \geq \frac{u\left(\overline{y}\right) - u\left(CE^u\right)}{u\left(y^h\right) - u\left(\overline{y}\right)}.$$

Replacing $\phi\left(u()\right)$ for $v()$ yields

$$\frac{\phi\left(u\left(\overline{y}\right)\right) - \phi\left(u\left(CE^v\right)\right)}{\phi\left(u\left(y^h\right)\right) - \phi\left(u\left(\overline{y}\right)\right)} \geq \frac{u\left(\overline{y}\right) - u\left(CE^u\right)}{u\left(y^h\right) - u\left(\overline{y}\right)}. \tag{19}$$

Since $\phi()$ is increasing and concave, and $u\left(y^h\right) > u\left(\overline{y}\right) > u\left(CE^u\right) > u\left(CE^v\right)$, we know that

$$\frac{\phi\left(u\left(\overline{y}\right)\right) - \phi\left(u\left(CE^v\right)\right)}{u\left(\overline{y}\right) - u\left(CE^u\right)} \geq \frac{\phi\left(u\left(\overline{y}\right)\right) - \phi\left(u\left(CE^u\right)\right)}{u\left(\overline{y}\right) - u\left(CE^u\right)} \geq \frac{\phi\left(u\left(y^h\right)\right) - \phi\left(u\left(\overline{y}\right)\right)}{u\left(y^h\right) - u\left(\overline{y}\right)}.$$

Rearranging yields (19). □

Proposition 2 means that we have the desirable comparative static result between risk aversion and the level of informal insurance, when complete insurance is achieved, using concavity of the utility function as the measure of risk aversion, and $1/\delta^*$ as the measure of the level of informal insurance. Proposition 2 is analogous to the well-known result that a more risk-averse agent is willing to pay more for formal, complete insurance, with the same measure of risk aversion.

In the case of formal insurance, we know that a decrease in wealth, or, equivalently, an increase in a lump-sum tax, makes risk-averse agents willing to pay more to avoid a given risk, if preferences exhibit nonincreasing absolute risk aversion (DARA). This comparative static result goes through to the informal insurance case as well, as the following corollary states.

Corollary 1. *If preferences are characterized by nonincreasing absolute risk aversion (DARA), then, in the complete insurance case, a decrease in wealth, or, an increase in a lump-sum tax, results in more informal insurance.*

Proof. Follows from Proposition 2 and the well-known result that, under DARA, a decrease in wealth is equivalent to an increasing and concave transformation of the utility function. □

Let us now turn to riskiness. A mean-preserving spread on the income distribution is taken as the criterion for ranking the riskiness of random incomes. I examine how $1/\delta^*$ changes when riskiness according to this standard concept changes under either of the following two assumptions.

Assumption (a). Income may take maximum three values.

Assumption (b). The support of the income distribution is constant.

Under assumption (a) and no aggregate uncertainty, there are only three possible income states: hl (agent 1 earning high income y^h, and agent 2 getting y^l) and lh (the reverse), and both occur with probability Pr^{asym}[8], and in third income state, both agents must earn $\overline{y} = \left(y^h + y^l\right)/2$.

[8] If, for example, agent i earned y^h with probability $\pi > Pr^{asym}$, agent $-i$ (the other agent) would get y^h with a smaller probability $1 - \pi < Pr^{asym}$, the two agents' expected incomes would differ, thus they would not be ex-ante identical.

To consider a mean-preserving spread in this case, let us define a new income distribution, $\widetilde{Y}$, as $\widetilde{y}^h = y^h + \epsilon$ and $\widetilde{y}^l = y^l - \epsilon$, with $\epsilon > 0$. Note that mean income does not change, that is, $\frac{\widetilde{y}^h + \widetilde{y}^l}{2} = \frac{y^h + y^l}{2} = \overline{y}$. In the third income state, nothing changes. Denote by $\widetilde{\delta}^*$ the corresponding discount factor above which perfect risk sharing is self-enforcing.

Under assumption (b), the extreme income realizations, y^h and y^l are kept constant, and the spread occurs on the "inside" of the distribution. Denote by $1/\widetilde{\delta}^*$, for this case as well, the level of informal insurance corresponding to the more risky income distribution, $\widetilde{Y}$.

Proposition 3. $1/\widetilde{\delta}^* \geq 1/\delta^*$*, that is, in the complete insurance case, under assumption (a) or (b), if income is riskier in the sense of a mean-preserving spread, then there is more informal insurance.*

Proof. Under assumption (a), (14) can be written as

$$\frac{1}{\delta^*} = 1 - Pr^{asym} + Pr^{asym} \frac{u(\overline{y}) - u\left(y^l\right)}{u(y^h) - u(\overline{y})}. \tag{20}$$

Thus, in this case, $1/\widetilde{\delta}^* \geq 1/\delta^*$ is equivalent to

$$\frac{u(\overline{y}) - u\left(\widetilde{y}^l\right)}{u(\widetilde{y}^h) - u(\overline{y})} \geq \frac{u(\overline{y}) - u\left(y^l\right)}{u(y^h) - u(\overline{y})}.$$

Replacing for $\widetilde{y}^h$ and $\widetilde{y}^l$ gives

$$\frac{u(\overline{y}) - u\left(y^l - \epsilon\right)}{u(y^h + \epsilon) - u(\overline{y})} \geq \frac{u(\overline{y}) - u\left(y^l\right)}{u(y^h) - u(\overline{y})}. \tag{21}$$

Now, since $u()$ is increasing and concave, we know that

$$\frac{u(\overline{y}) - u\left(y^l - \epsilon\right)}{\overline{y} - y^l + \epsilon} \geq \frac{u(\overline{y}) - u\left(y^l\right)}{\overline{y} - y^l},$$

and

$$\frac{u\left(y^h + \epsilon\right) - u(\overline{y})}{y^h + \epsilon - \overline{y}} \leq \frac{u\left(y^h\right) - u(\overline{y})}{y^h - \overline{y}}.$$

Then, using the fact that $y^h - \overline{y} = \overline{y} - y^l$, dividing gives (21).

Under assumption (b), $1/\widetilde{\delta}^* \geq 1/\delta^*$ is equivalent to

$$\frac{u(\overline{y}) - u\left(\widetilde{CE}^u\right)}{u(y^h) - u(\overline{y})} \geq \frac{u(\overline{y}) - u(CE^u)}{u(y^h) - u(\overline{y})}, \tag{22}$$

where $\widetilde{CE}^u$ is the certainty equivalent of the riskier distribution $\widetilde{Y}$. It is well known that $\widetilde{CE}^u < CE^u$, thus (22) holds. □

Thus, in the complete insurance case, $1/\delta^*$ is consistent with a mean-preserving spread as the measure of riskiness, assumptions (a) or (b) being sufficient conditions. Now, let us consider second-order stochastic dominance (SSD) as the measure of riskiness. With a constant mean, the above result naturally extends to SSD, since an SSD deterioration is equivalent to a sequence of mean-preserving spreads. The result still holds if the dominated process has a lower mean, as the following corollary states.

Corollary 2. *In the complete insurance case, under assumption (b), if income is riskier in the sense of an SSD deterioration, then there is more informal insurance.*

Proof. Follows from the proof of Proposition 3, noting that if $\widetilde{Y}$ is dominated by Y in the sense of SSD, then $\widetilde{CE}^u < CE^u$ for any $u()$ increasing and concave. □

Thus assumption (b) is a sufficient condition for the desirable comparative static result, using SSD to compare the riskiness of income distributions. Future work should determine necessary conditions.

5 The incomplete insurance case

This section examines the case where agents can only achieve *incomplete insurance*, even though they share risk perfectly. In this case, agents must bear some consumption risk. This is equivalent to a setting where a community faces aggregate risk as well, while agents can only provide insurance to each other against idiosyncratic risks. That is, I assume that income is realized independently for the two agents. Remember that to have no aggregate uncertainty, as in section 4, income realizations have to be perfectly negatively correlated across agents. As in the standard insurance setting when the agent cannot buy complete insurance, one may also say that there is background risk. I am interested in what goes through from the results of section 4. First, I look at risk aversion, then riskiness considering a mean-preserving spread.

Let us consider risk aversion first. Remember that $u()$ and $v()$ are two utility functions, and we have assumed that an agent with utility function $v()$ is more risk averse than an agent with utility function $u()$. That is, there exists an increasing and concave function $\phi()$, such that $v() = \phi(u())$. Remember also that δ_v^* (δ_u^*) denotes the discount factor above which perfect risk sharing is self-enforcing, if agents have utility function $v()$ ($u()$). The following assumption is sufficient to guarantee that the desirable comparative static result holds.

Assumption (c). $u(y_1(s)) + u(y_2(s)) \leq 2u(\overline{y})$, where $\overline{y} = \frac{y^h + y^l}{2}$, for all s where $y_1(s) \neq y_2(s)$.

This assumption means that there is no asymmetric state where the expected utility in autarky would be higher than the utility from consuming $\overline{y}$.

Proposition 4. *In the incomplete insurance case, under assumption (c), $1/\delta_v^* \geq 1/\delta_u^*$. That is, if agents are more risk averse in the sense of having a more concave utility function, then more informal insurance is achieved.*

Proof. Using the formula determining $1/\delta^*$, equation (14), for $1/\delta_v^* \geq 1/\delta_u^*$ to hold it is sufficient that

$$\frac{2v\left(\frac{y_1(s)+y_2(s)}{2}\right) - v(y_1(s)) - v(y_2(s))}{v(y^h) - v(\overline{y})} \geq \frac{2u\left(\frac{y_1(s)+y_2(s)}{2}\right) - u(y_1(s)) - u(y_2(s))}{u(y^h) - u(\overline{y})}, \forall s. \quad (23)$$

Denote by $Ey(s)$ mean income at state s, and by $CE^u(s)$ the certainty equivalent at state s, when preferences are described by the utility function $u()$, that is, $u(CE^u(s)) = \frac{1}{2}u(y_1(s)) + \frac{1}{2}u(y_2(s))$. Then, (23) can be written as

$$\frac{v(Ey(s)) - v(CE^v(s))}{v(y^h) - v(\overline{y})} \geq \frac{u(Ey(s)) - u(CE^u(s))}{u(y^h) - u(\overline{y})}.$$

To complete the proof, one may use the same argument as in the proof of Proposition 2. □

Here I put a restriction on the income process that is a sufficient condition for more risk aversion to increase voluntary insurance. One could also follow another approach, like Ross (1981) in the case of formal insurance, to find a stronger measure of risk aversion.

Let us now turn to riskiness, in particular, how $1/\delta^*$ changes if there is a mean-preserving spread on the income distribution. I provide counterexamples to the expected comparative static result. It turns out to be sufficient to examine the simplest possible setting. Thus, suppose that income may only take two values, high or low, denoted y^h and y^l, respectively, with $y^h > y^l > 0$. Let π denote the probability of earning y^h. Remember that hl and lh are the asymmetric states (agent 1 gets y^h, while agent 2 earns y^l, or the reverse). Then $Pr^{asym} = \pi(1-\pi)$.

Now, let us define a new, more risky income distribution, $\widehat{Y}$, in the sense of a mean-preserving spread. Let the new high income realization be $\widehat{y}^h = y^h + \epsilon$, with $\epsilon > 0$. To keep mean income constant, $\widehat{y}^l$ must equal $y^l - \frac{\pi}{1-\pi}\epsilon$, with $\epsilon < \frac{1-\pi}{\pi}y^l$. Note that in this case consumption in the asymmetric states is $\frac{\widehat{y}^h+\widehat{y}^l}{2} = \frac{y^h+y^l}{2} + \frac{1-2\pi}{1-\pi}\frac{\epsilon}{2}$. Denote the corresponding level of informal insurance by $1/\widehat{\delta}^*$.

Proposition 5. *It is not true in general that* $1/\widehat{\delta}^* \geq 1/\delta^*$. *That is, with incomplete insurance, a mean-preserving spread on incomes may result in less informal insurance.*

Proof. Let us construct a counterexample. Take $y^h = 1.5$, $y^l = 0.55$, $\pi = 0.6$ (so mean income is $0.6 \cdot 1.5 + 0.4 \cdot 0.55 = 1.12$), thus $Pr^{asym} = \pi(1-\pi) = 0.6 \cdot 0.4 = 0.24$, and $\epsilon = 0.2$. It follows that $\frac{y^h+y^l}{2} = 1.025$, and $\widehat{y}^h = 1.7$, $\widehat{y}^l = 0.25$, and $\frac{\widehat{y}^h+\widehat{y}^l}{2} = 0.975$. The mean is now $0.6 \cdot 1.7 + 0.4 \cdot 0.25 = 1.12$. Thus the distribution $\widehat{Y}$ is indeed a mean-preserving spread of Y. Consider the utility function

$$u(c) = \begin{cases} c^{0.8} & \text{if } c < 1 \\ c^{0.1} & \text{if } c > 1 \end{cases},$$

and smooth it appropriately in a small neighborhood of 1. This utility function could represent the preferences of a loss-averse agent. Replacing the above values in (15), we have

$$\frac{1}{\delta^*} = 1 - 0.24 + 0.24\frac{1.025^{0.1} - 0.55^{0.8}}{1.5^{0.1} - 1.025^{0.1}} = 3.12,$$

and

$$\frac{1}{\widehat{\delta}^*} = 1 - 0.24 + 0.24\frac{0.975^{0.8} - 0.25^{0.8}}{1.7^{0.1} - 0.975^{0.8}} = 2.85,$$

which contradicts $1/\widehat{\delta}^* \geq 1/\delta^*$.

The result does not hinge on the fact that $\pi > \frac{1}{2}$, and that therefore consumption in the asymmetric states, hl and lh, decreases. Take $y^h = 1.5$, $y^l = 0.495$, $\pi = 0.1$, and $\epsilon = 0.6$, and consider the same utility function as above. This specification provides another counterexample, since $1/\delta^* = 1.84$ and $1/\widehat{\delta}^* = 1.35$. $\square$

The intuition behind this result is the following. In the case of incomplete insurance, when income becomes riskier in the sense of a mean-preserving spread, not only the spread between the high and low income realizations changes, but also consumption in the asymmetric states. As a result, the transfers $\frac{y^h+y^l}{2} - y^l = y^h - \frac{y^h+y^l}{2}$ are not just increased to $\frac{y^h+y^l}{2} - y^l + \frac{1}{1-\pi}\frac{\epsilon}{2} = y^h - \frac{y^h+y^l}{2} + \frac{1}{1-\pi}\frac{\epsilon}{2}$, but they also occur at consumption levels that are shifted by $\frac{1-2\pi}{1-\pi}\frac{\epsilon}{2}$ at the mean. Because of this shift, the utility gain of insurance represented by $u\left(\frac{y^h+y^l}{2}\right) - u\left(y^l\right)$, and the loss of insurance represented by $u\left(\frac{y^h+y^l}{2}\right) - u\left(y^h\right)$ are evaluated at a different consumption level for the income distribution Y than for $\widehat{Y}$. The curvature of the utility function may differ sufficiently at the two consumption levels, so that the ratio between the utility gain and loss of informal insurance changes in an ambiguous way, when a mean-preserving spread occurs on the income distribution. In particular, the level of informal insurance may decrease.

This result points out that, when agents share risk informally, determining how much consumption variability they have to deal with is a rather complex issue, since the link between income risk, in some standard sense, and consumption risk is not straightforward. This is the consequence of the interplay of idiosyncratic and aggregate risk. See also Attanasio and Ríos-Rull (2000), who show that aggregate insurance may reduce welfare, when agents share (idiosyncratic) risk informally.

How to reconcile this negative result? Aggregate risk should be kept constant, while idiosyncratic risk increases. To do this, some negative correlation between the income realizations of the two agents has to be reintroduced. This can indeed work, as the following example demonstrates.

Example. Let us reconsider the first example of the proof above. The original income distribution, Y, was $y^h = 1.5$, $y^l = 0.55$, with the probability of the high income realization

$\pi = 0.6$, that is, $Pr^{asym} = 0.24$. The second, more risky income distribution, $\widehat{Y}$, was $\widehat{y}^h = 1.7$, $\widehat{y}^l = 0.25$, with $\pi = 0.6$ still. Expected individual income is 1.12 for both agents, while expected aggregate income is 2.24 for both income distributions. We wanted to increase idiosyncratic risk, however, aggregate risk has also increased. In particular, the standard deviation of the distribution of aggregate income has increased from 0.5472 to 0.8352.[9] Now, let us introduce some negative correlation between the income realizations of the two agents for $\widehat{Y}$, to match the standard deviation of Y. This can be achieved my setting $Pr^{asym} = 0.364$, and decreasing the probability of the hh and ll states by 0.124 each. Let us denote the level of informal insurance by $1/\check{\delta}^*$ in this case. Then $1/\delta^* = 3.12$ as before, but $1/\check{\delta}^* = 3.81$, thus, keeping aggregate risk constant, informal insurance increases as a result of a mean-preserving spread on the income distribution.

6 Discussion on measuring risk

This paper has looked at how informal insurance is related to risk aversion of agents and the riskiness of the distribution from which their income is drawn. Part of the motivation for this paper was to reconsider what would be a good measure of risk that agents want to insure against. Thus this section reconsiders riskiness, first by some comments on the literature, then by relating riskiness to informal insurance and $1/\delta^*$.

There is reasonable consensus on what risk, or uncertainty, is. Further, it is widely accepted to assume that agents dislike *pure*, or zero-mean risks, in other words, agents are risk averse. However, to say that a gamble, or a stochastic process, is *more risky* than another, or, to say that an agent is *more risk averse* than another, is much more complicated. An additional question is, how we can disentangle risk from the expected value of the process. Further, can we talk about the riskiness of a distribution, or a risk faced by an agent, without actually taking into account her preferences with respect to risk?

The first issue is measuring risk aversion. A complete, but local measure is proposed by Arrow (1965) and Pratt (1964). The Arrow-Pratt coefficient of absolute risk aversion is

[9] Note that speaking about the standard deviation or the coefficient of variation is equivalent here, since the mean doesn't change.

widely used, and has some useful properties. Namely, if agent i is more risk averse in the sense of Arrow-Pratt than agent j, then (i) i will invest less in the risky asset in a simple two-asset portfolio model, and (ii) i will be willing to pay more for insurance against having to face some given gamble. However, if both assets are risky, or only incomplete insurance is available against a gamble, in other words, there is background risk, the Arrow-Pratt measure no longer has nice properties. Ross (1981) defines a somewhat stronger measure of risk aversion, that has the desirable properties. See also Jewitt (1987) on the conditions for the comparative static results (i) and (ii) in the presence of background risk, keeping the Arrow-Pratt measure of risk aversion, but putting stronger restrictions on the stochastic processes.

Brachinger (2002) reviews the methods for measuring risk purely based on the stochastic process, and not taking into account preferences. As simple measures of riskiness, the variance has been widely used. For example, in the Capital Asset Pricing Model (CAPM) the riskiness of the market portfolio is given by volatility, that is, the standard deviation of returns. Some rather complicated measures have also been developed, taking into account, for example, the difference between gains and losses.

However, the most well-know measure of the riskiness of a random variable is probably second-order stochastic dominance (SSD), due to Hadar and Russell (1969), and Rothschild and Stiglitz (1970). This measure is directly related to risk preferences. The random variable Y dominates variable Z in the sense of second-order stochastic dominance if and only if *all* risk-averse agents prefer Y to Z. Two things are important to note about SSD. First, it is a partial order, and presents a strong requirement. Second, it is not really a measure of riskiness, rather of the desirability of a random variable. Note that the dominated distribution Z cannot have a higher mean.

Jewitt (1989) develops a way, based on SSD, to measure the riskiness of processes with different means. He derives comparative static results related to the concavity of the utility function, and the choice between two risky distributions. One of the aims of the present paper is similar, but instead of considering the choice between gambles, it examines informal insurance to attenuate the adverse effects of risk.

In a recent paper, Aumann and Serrano (2006) take *duality* between risk preferences and riskiness as an axiom. They argue that riskiness should be measured in a way that more risk-averse individuals would accept less risky gambles. They come up with a new measure, in particular, the reciprocal of the coefficient of absolute risk aversion of an individual who is indifferent between taking or not taking the gamble. This measure satisfies duality and other axioms. The authors restrict their attention to one type of preferences, namely, those characterized by constant absolute risk aversion (CARA). If, by contrast, agents have nonincreasing absolute risk aversion (DARA), the empirically more plausible case, the proposed measure loses its nice properties. Another problem with this approach, shared with SSD, is that expected value and riskiness are confounded. The authors argue that, when one distribution yields a higher payoff in all states of the world, then it cannot be considered riskier.

Consider two simple distributions, Y and Z, that both have two possible realizations, high or low, determined by the toss of a fair coin. Y yields 1 or 2 euros, while Z givess 3 or 100. SSD or the measure of Aumann and Serrano (2006) tell us that Y is more risky, however, Z seems to involve more variation, or risk.

Relating riskiness and risk aversion to informal insurance makes their inherent interdependence clear. As a consequence, a measure of riskiness has to take into account the risk preferences of agents. The present paper considers risk to be inherently related to risk preferences (duality axiom), but aims to disentangle risk and expected value. Risk can be considered as something that agents would like to and can insure against, unlike expected value. $1/\delta^*$ also satisfies duality in the sense that, given the random variable Y, it is able to rank agents' preferences in terms of how risk averse they are.

In the case of CARA (CRRA) preferences, we have seen that $1/\delta^*$ measures risk consistently with the standard deviation (coefficient of variation). Considering $1/\delta^*$ to judge the riskiness of a distribution, given risk preferences, may be a way to disentangle the effects of expected value and riskiness in general. This is in contrast to measures like second-order stochastic dominance (SSD), where expected value and risk are confounded, thus SSD measures rather the desirability of the random variable than its riskiness. A desirable property of $1/\delta^*$ is that it is a scalar measure, and it gives a complete ordering of riskiness, given some utility function, for distributions with any mean.

7 Conclusion

This paper has shown a way to characterize the amount of voluntary insurance achieved between agents, and made a first attempt to relate it to riskiness and risk aversion. In particular, I defined the level of informal insurance, denoted $1/\delta^*$, as the reciprocal of the discount factor above which perfect risk sharing is self-enforcing.

This paper then performed some comparative statics exercises to see how $1/\delta^*$ changes, when risk aversion or the riskiness of the income process changes in some standard sense. Results include that, if the utility function is more concave, that is, agents are more risk averse, $1/\delta^*$ is higher. However, in the case where insurance is incomplete, that is, when agents cannot smooth their consumption completely even when sharing risk perfectly, a mean-preserving spread on the income process may result in less informal insurance. This is because of the interplay of idiosyncratic and aggregate risk. This negative result can be reconciled, if aggregate risk is kept constant.

It is an interesting task to future research to determine both sufficient and necessary conditions on the the utility function under which more informal insurance is achieved, when a mean-preserving spread, or, more generally, a second-order stochastic deterioration occurs. One could also find restrictions for the random variable Y.

The special cases of CARA and CRRA preferences were also examined in detail. I found that $1/\delta^*$ only depends on risk aversion and the riskiness of the income process as measured by the standard deviation (the coefficient of variation), in the case of CARA (CRRA) preferences, and is independent of expected income. Therefore, in line with existing literature, one may say that the standard deviation (coefficient of variation) is an appropriate measure of riskiness in the CARA (CRRA) case, relating riskiness to informal insurance.

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8 Appendix

Proof of Claim 2:

Replace the utility function (17) and $y^h = y$ and $y^l = (1-q)\,y$ in the equation determining $1/\delta^*$ in the case of two possible income realizations, equation (20). For $\sigma \neq 1$, this gives

$$\frac{1}{\delta^*} = \frac{1}{2} + \frac{1}{2}\frac{\frac{\left(\left(1-\frac{q}{2}\right)y\right)^{1/\sigma}}{1-\sigma} - \frac{((1-q)y)^{1/\sigma}}{1-\sigma}}{\frac{y^{1-\sigma}}{1-\sigma} - \frac{\left(\left(1-\frac{q}{2}\right)y\right)^{1/\sigma}}{1-\sigma}},$$

which can be rewritten as

$$\frac{1}{\delta^*} = \frac{1}{2}\frac{(1-q)^{1-\sigma} - 1}{\left(1-\frac{q}{2}\right)^{1-\sigma} - 1}. \tag{24}$$

For $\sigma = 1$, we have $1/\delta^* = ln\,(1-q)\,/\left(2ln\left(1-\frac{q}{2}\right)\right)$. Thus $1/\delta^*$ only depends on σ and q, and is independent of y, that is, of mean income.

Now, I want to show that $\frac{\partial 1/\delta^*(\sigma,q)}{\partial\sigma} > 0$ and $\frac{\partial 1/\delta^*(\sigma,q)}{\partial q} > 0$. Let us suppose that $\sigma \neq 1$. The results generalize to $\sigma = 1$ taking limits. Let us differentiate equation (24) with respect to σ first. This gives

$$\begin{aligned}
sign\left(\frac{\partial 1/\delta^*(\sigma,q)}{\partial\sigma}\right) &= sign\left(\frac{1}{2}\left[(1-q)^{1-\sigma}\,ln\,(1-q)\,(-1)\left(\left(1-\frac{q}{2}\right)^{1-\sigma} - 1\right) - \right.\right. \\
&\left.\left. - \left(1-\frac{q}{2}\right)^{1-\sigma} ln\left(1-\frac{q}{2}\right)(-1)\left((1-q)^{1-\sigma} - 1\right)\right] / \left(\left(1-\frac{q}{2}\right)^{1-\sigma} - 1\right)^2\right) \\
&= sign\left(ln\left(1-\frac{q}{2}\right)\left(1-(1-q)^{\sigma-1}\right) - ln\,(1-q)\left(1-\left(1-\frac{q}{2}\right)^{\sigma-1}\right)\right) \\
&= sign\left(\frac{1-(1-q)^{\sigma-1}}{ln\,(1-q)} - \frac{1-\left(1-\frac{q}{2}\right)^{\sigma-1}}{ln\left(1-\frac{q}{2}\right)}\right),
\end{aligned}$$

where the third line follows after dividing by $(1-q)^{1-\sigma}\left(1-\frac{q}{2}\right)^{1-\sigma} > 0$, and the last line follows dividing by $ln\,(1-q)\,ln\left(1-\frac{q}{2}\right) > 0$. We know that $0 < \frac{q}{2} < q < 1$, thus $0 < 1-q < 1-\frac{q}{2} < 1$. What remains to be shown is that the function

$$f(z) \equiv \frac{1-z^{\sigma-1}}{ln(z)}$$

is decreasing in z, $z \in (0,1)$. To do this, let us differentiate $f(z)$ with respect to z. This gives

$$
\begin{aligned}
sign\left(\frac{\partial f(z)}{\partial z}\right) &= sign\left(\frac{-(\sigma-1)\,z^{\sigma-2}ln(z)-\frac{1}{z}\left(1-z^{\sigma-1}\right)}{\left(ln(z)\right)^{2}}\right)\\
&= sign\left(\left(1-(\sigma-1)\,ln(z)\right)z^{\sigma-1}-1\right).
\end{aligned}
$$

Note that $\lim_{z\to1}\left(1-(\sigma-1)\,ln(z)\right)z^{\sigma-1}-1=0$. Now, to show that $\left(1-(\sigma-1)\,ln(z)\right)z^{\sigma-1}-1<0$, we only have to establish that $g(z)\equiv\left(1-(\sigma-1)\,ln(z)\right)z^{\sigma-1}-1$ is increasing in z for $z\in(0,1)$. Taking derivatives with respect to z gives

$$
\begin{aligned}
sign\left(\frac{\partial g(z)}{\partial z}\right) &= sign\left((\sigma-1)\,z^{\sigma-2}-(\sigma-1)\left(\frac{1}{z}z^{\sigma-1}+ln(z)\,(\sigma-1)\,z^{\sigma-2}\right)\right)\\
&= sign\left(-(\sigma-1)^{2}\,ln(z)\,z^{\sigma-2}\right).
\end{aligned}
$$

The first term is positive, the second is negative, the third is positive, and all this is multiplied by (-1), thus $\frac{\partial g(z)}{\partial z}$ is positive. It follows that $\frac{\partial f(z)}{\partial z}$ is negative, and that $\frac{\partial 1/\delta^{*}(\sigma,q)}{\partial\sigma}$ is positive.

Now, let us differentiate equation (24) with respect to q. This gives

$$
\begin{aligned}
sign(\frac{\partial 1/\delta^{*}(\sigma,q)}{\partial q}) &= sign\left(\frac{1}{2}(1-\sigma)\left[(1-q)^{-\sigma}(-1)\left(\left(1-\frac{q}{2}\right)^{1-\sigma}-1\right)-\right.\right.\\
&\left.\left.-\left((1-q)^{1-\sigma}-1\right)\left(1-\frac{q}{2}\right)^{-\sigma}\left(-\frac{1}{2}\right)\right]/\left(\left(1-\frac{q}{2}\right)^{1-\sigma}-1\right)^{2}\right)\\
&= sign\left((1-\sigma)\left[\frac{1}{2}\left((1-q)^{1-\sigma}-1\right)\left(1-\frac{q}{2}\right)^{-\sigma}-\right.\right.\\
&\left.\left.-(1-q)^{-\sigma}\left(\left(1-\frac{q}{2}\right)^{1-\sigma}-1\right)\right]\right)\\
&= sign\left((1-\sigma)\left[\left(1-\frac{q}{2}\right)^{\sigma}-\frac{1}{2}\left((1-q)^{\sigma}+1\right)\right]\right).
\end{aligned}
$$

The last line follows after dividing by $(1-q)^{-\sigma}>0$ and $\left(1-\frac{q}{2}\right)^{-\sigma}>0$. We have to consider two cases.

- $\sigma<1$. Now $1-\sigma>0$, so we have to show that $\left(1-\frac{q}{2}\right)^{\sigma}-\frac{1}{2}\left((1-q)^{\sigma}+1\right)>0$.
- $\sigma>1$. In this case $1-\sigma<0$, so we have to show that $\left(1-\frac{q}{2}\right)^{\sigma}-\frac{1}{2}\left((1-q)^{\sigma}+1\right)<0$.

We may rewrite this last expression as

$$
\left(1-\frac{q}{2}\right)^{\sigma}-\frac{1^{\sigma}+(1-q)^{\sigma}}{2}. \tag{25}
$$

Note that $1-\frac{q}{2}$ is the mean of 1 and $1-q$. Let us define $h(z) \equiv z^{\sigma}$. So what we are comparing is the mean (a convex combination) of the values $h(1)$ and $h(1-q)$ to the value at the mean, that is, $h\left(\frac{1+1-q}{2}\right) = h\left(1-\frac{q}{2}\right)$.

- $\sigma < 1$. $h(z)$ is concave, thus $h\left(1-\frac{q}{2}\right) > \frac{h(1)+h(1-q)}{2}$. It follows that (25) is positive, as I wanted to show.

- $\sigma > 1$. $h(z)$ is convex, thus $h\left(1-\frac{q}{2}\right) < \frac{h(1)+h(1-q)}{2}$. It follows that (25) is negative, and this is what I wanted to show.