

Price Rigidity and Strategic Uncertainty

An Agent-based Approach

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Abstract. The phenomenon of infrequent price changes has troubled economists for decades. Intuitively one feels that for most price-setters there exists a range of inaction, i.e. a substantial measure of the states of the world, within which they do not wish to modify prevailing prices. However, basic economics tells us that when marginal costs change it is rational to change prices, too. Economists wishing to maintain rationality of price-setters resorted to fixed price adjustment costs as an explanation for price rigidity. In this paper we propose an alternative explanation, without recourse to any sort of physical adjustment cost, by putting strategic interaction into the center-stage of our analysis. Price-making is treated as a repeated oligopoly game. The traditional analysis of these games cannot pinpoint any equilibrium as a reasonable "solution" of the strategic situation. Thus there is genuine strategic uncertainty, a situation where decision-makers are uncertain of the strategies of other decision-makers. Hesitation may lead to inaction. To model this situation we follow the style of agent-based models, by modelling firms that change their pricing strategies following an evolutionary algorithm. Our results are promising. In addition to reproducing the known negative relationship between price rigidity and the level of general inflation, our model exhibits several features observed in real data. Moreover, most prices fall into the theoretical "range" without explicitly building this property into strategies.

1 Introduction

Everyday observations tell us that some prices are changing almost continuously. Financial asset prices, of foreign exchange or of stocks, are obvious examples for each of us. While most people do not buy "commodities" regularly, it is also well-known that the prices of crude oil, gold, or grain behave similarly. On the other hand most prices we meet in shops or kiosks seem familiar, we expect that they do not change from one day to another.

Infrequent price or wage (the price of labor) changes have troubled macroeconomists. According to the prevailing wisdom this fact is behind the ability of monetary policy to affect (significantly) real - as opposed to merely nominal

- variables. It is perhaps less in evidence but the "no change in prices" phenomenon is a concern for competition offices, too, as it may signal collusion.

Intuitively one feels that for most price-setters there exists a range of inaction, i.e. a substantial "measure of states of the world" where they do not wish to modify prevailing prices. However, basic economics tells us that at least when marginal costs change it is rational to change prices, too, even in not fully competitive markets. However, many prices do not even react to clearly visible marginal cost variations, in macroeconomics the perhaps most salient example of this is the PPP puzzle (see Rogoff 1996), i.e the observation that there are large variations across currencies in purchasing power parity induced by nominal exchange rate changes.

Economists wishing to maintain rationality of price-setters resorted to fixed price adjustment costs (menu costs) as an explanation for price rigidity. The fixed adjustment costs story is plausible, but only partly. The everyday reprinting of an elegant menu for a restaurant is really not an easily digestible option. However, what about meal prices written on a blackboard at the entrance? Or what about magazine prices, one of the foremost examples of unreasonably strong price inflexibility? Here printing a different price on each new edition would not entail even a negligible extra cost.

It must be the case that the rigidity of prices has more than one reason. In this paper we propose an alternative explanation, without recourse to any sort of physical adjustment cost. We put strategic interaction into center-stage in our analysis. While repeated oligopolistic pricing games can produce "collusive" equilibria with rigid prices in peculiar circumstances (see Athey-Bagwell-Sanchirico 2010), we focus on the possible role of strategic uncertainty.

The main idea of the approach can be summarized as follows. Price-making is a repeated game, firms usually act in markets where there exist identifiable competitors. The traditional analysis of these games cannot pinpoint any equilibrium as a reasonable "solution" of the strategic situation. Thus there is genuine strategic uncertainty, a situation where decision-makers are uncertain of the strategies of other decision-makers. Strategic uncertainty may cause hesitation: if I cut the price would it be interpreted as a signal for a "price-war"? or if I raise the price would it be followed by others, or shall I lose market-share? Hesitation may lead to inaction, as we all know too well. To model this situation we follow the style of agent-based models. Agent-based models can accommodate a wider range of "environments" than the traditional approaches (see below in detail). The definition of strategies is a common problem to all approaches, but in this respect agent-based models can exhibit larger flexibility, too. A significant difference is that whereas traditional approaches rely on either full rationality and an equilibrium concept, or a specific form of bounded rationality and an equilibrium

concept, agent-based models must assume something about learning.

In Section 2 we give a short survey of the price rigidity literature, followed by some notes on agent-based modelling applied to economics problems. In the following section we theoretically motivate our approach, and in Section 4 the agent-based oligopoly model is set up, and the learning algorithm is discussed. Section 5 presents the baseline scenario, while Section 6 offers sensitivity analyses. The concluding section summarizes, pointing out paths to further research.

2 Literature Survey

2.1 The Rigidity of Prices

Price rigidity is basic ingredient of new keynesian macroeconomic models. New-Keynesian Phillips curves are derived from individual profit maximization, whereby prices are set by monopolistically competitive firms that face costs of price adjustment. Several types of price adjustment costs models have been proposed in the literature. The aggregate implications of these models are similar, though in no case identical. (See for example Roberts 1995.) Researchers have frequently shown a lack of concern for looking for the "right" pricing model, as long as the implications seemed to be in line with macroeconomic data, and one should say, *a priori* beliefs.

However, in the last decade a series of systematic studies was launched to find out from microdata how firms "in reality" set their prices. (This literature is surveyed in Mackowiak-Smets 2008.) One of the studies (Fabiani et al. 2007) was conducted by nine Eurosystem central banks, through standardized surveys. It covered more than 11,000 firms. Of the conclusion we cite only a few pertinent to our paper. First: "The results, robust across countries, show that firms operate in monopolistically competitive markets, where prices are mostly set following markup rules ..." Though the authors talk of "monopolistically competitive markets", we believe that the evidence cannot really distinguish between "monopolistic competition" and oligopoly. What is important for us is the prevalence of markup rules. Second: "Price stickiness is mainly driven by customer relationships – explicit and implicit contracts – and coordination failure." We do not address customer relationship, but emphasize "coordination failure". Indirectly this sentence vindicates our claim with respect to oligopolistic competition. To wit: there is no reason for coordination in monopolistic competition. This is the source of price rigidity we wish to make sense of in this study. The microeconomic research has in general had a dim conclusion for adjustment costs models: each of them seems to be irreconcilable with some salient features of microdata. Third: "Firms facing high competitive pressures review their prices more frequently." We interpret this finding that it means that more firms, other things being equal, would care less for solving the coordination problem, and, therefore, would review and possibly change their prices more frequently. It is an interesting question to ask what makes prices change if anything. Papers that

have addressed this issue usually found that cost changes are more likely to be responsible for price change than variation in demand. (See Bils-Klenow 2004.)

As a response to the failure of adjustment cost models to account for the microeconomic evidence Rotemberg (2010) proposed an alternative in the behavioral economics style. Essentially Rotemberg tried to make intelligible the concept of fair pricing. His analytical result is that a concern for fairness (partly because customers can be offended by unfair behavior by firms) can cause price rigidity.

Recent research has yielded quantitative results as well. Several authors calculated price-rigidity statistics for different time-periods, and areas. On a consumer-price data base for the US Klenow-Krystov (2008) obtained that 36 % of all prices are changed in every month, and that the mean duration of prices is 6.9 months. More substantial price rigidity was found by Dhyne et al. (2005) for the Euroregion. The respective statistics are 15 %, and 13 months. For a higher inflation country Bauer (2008) examined Hungarian retail food prices, and he found 25 %, and 3.8 months. With broader coverage also for Hungary Gabriel and Reiff (2008) reached estimates of 21.5 %, and of 8 months. These studies have found substantial heterogeneity across sectors, and as expected, a negative relationship between the level of overall inflation and the degree of price rigidity.

Eichenbaum, Jaimovich and Rebelo (2008) analysed the price and cost data of a large American chain store retailer. They found substantial rigidity for "reference" prices, remaining unchanged on average for a year. Actual prices are moving more, but this flexibility must be a feature of the concrete market segment, and must not concern us here. What is important for us is the finding that reference prices are adjusted whenever they differ significantly from a target price defined as average cost times a "required" markup. The authors calculate the tolerance level as 20 %. Below we will model exactly this type of pricing strategy.

2.2 Agent-based Modelling

Agent-based models have been used for some time in economics. General surveys are available, such as Tesfatsion (2001, 2006), LeBaron (2006), or see Heath-Hill-Ciarallo (2009) from the methodological point of view. All of these approaches share a bounded rationality philosophy, wherein agents apply strategies that do not maximize necessarily anything, and are limited in their understanding of their environment. Their decisions are also "computable", excluding thereby behaviours unrealizable by a computer. Agent-based models are simulated, which implies that the interaction of the agents is also well-defined and computable, not admitting, for instance, equilibrium prices that merely satisfy certain conditions but even the modeller could not calculate them.

We are especially interested in agent-based models that allow for an endogenous updating of strategies, which is usually styled as learning. Learning models have also a long history in economics, see for a survey Evans-Honkapohja (1999). Learning, specifically, in the agent-based setting was reviewed by Brenner (2006) and Duffy (2006). Though many approaches to learning exist one of them draw particular attention in economics applications: evolutionary learning. (For a survey see Arifovic 2000). One can classify as of evolutionary learning algorithms the genetic algorithm, genetic programming, the classifier systems. We opted for this general approach on the ground that it has proved to be relatively successful in several economic applications.

Oligopolistic markets have been studied by agent-based modelling. Midgley et al. (1997) is an example that addresses the problem from an operations-research perspective. The paper whose approach is the closest to ours is Chen-Ni (2000). Here the authors explore the "ecology" generated by evolutionary learning in repeated oligopoly games. Their focus is on the "cooperation vs price wars" issue, they constrain the set of feasible prices to high (cooperative) and low (punishing). Thus their findings are irrelevant to the issue of price rigidity.

3 Motivation: A Symmetric Pricing Game

There are I ($i = 1, \dots, I$) firms, $p = (p_i, p_{-i})$ is the price vector. The per-period profit functions are given:

$$\pi_i(x, p_i, p_{-i})$$

The profit functions are identical, for all i , and with identical prices they can be written as $\pi(x, p)$, where x is a state exogenous to the market.

Let $p^N(x)$ the static symmetric unique Nash-equilibrium price vector at x , and $\pi^N(x)$ the associated payoff. Let us suppose also that the best reply functions are increasing in competitors' prices.

Let

$$V^N(x_t) = \pi^N(x_t) + \beta E_t \pi^N(x_{t+1}) + \beta^2 E_t \pi^N(x_{t+2}) + \dots + \beta^n E_t \pi^N(x_{t+n}) + \dots$$

be the dynamic Nash-payoff at x_t .

The state vector for an individual firm is (x_t, p_{t-1}) . The strategy $(p^N(x_t))$ is a subgame perfect equilibrium in this game, and can serve as a reference point (the worst equilibrium) for the oligopolists.

If the firms can always tacitly collude they can implement the first best collusive strategy $(p^{CFB}(x_t))$ that maximizes industry (and per firm) profits at each

instant. Depending on β repeated game theory establishes whether ($p^{CFB}(x_t)$) can be made the outcome of a subgame perfect equilibrium or not. Also it is a normal feature of repeated games that many outcomes between ($p^N(x_t)$) and ($p^{CFB}(x_t)$) are also available as subgame perfect equilibrium paths, depending on beliefs and out of equilibrium strategies.

Suppose there exists a specific subgame perfect equilibrium, that involves a region of inaction, i.e. circumstances under which prices are not changed, despite changes in x_t . The basic intuition is the following: at each instant (period) there exists a highest implementable collusive price $p_t^C > p^N(x_t)$, which, if chosen by all firms, gives each a higher payoff than the Nash-payoff. This price, which we will call for simplicity the collusive price, is not necessarily the same as $p^{CFB}(x_t)$, and must be determined endogenously. The prevailing prices (p_{t-1}) can fall into three regions.

1. $p_{t-1} < p^N(x_t)$. Here firms can successfully collude and jump to the collusive price, p_t^C . The reason is that all of them have to move "upwards", and no one equivocates with respect to others' behaviour, or others' beliefs about others' behaviour.
2. $p^N(x_t) < p_{t-1} < p_t^C$. This is the region of inaction. Each firm's myopic best response to the prevailing prices is to decrease its own price (going closer to the Nash-price), whereas collective rationality would dictate a coordinated increase in prices. Uncertainty engenders inaction since: if I increase the price unilaterally I will lose, and if I decrease the price I'm afraid it will result in a price war, ending up in Nash-behaviour.
3. $p_t^C < p_{t-1}$. Successful collusion again, as both myopic individual rationality, and cooperative wisdom calls for decreasing the prices.

The problem formulated in this way admits of the application of dynamic programming. Therefore, if $p^N(x_t) < p_{t-1} < p_t^C$ then $p_t = p_{t-1}$, and the value function:

$$V(x_t, p_{t-1}) = \pi(x_t, p_{t-1}) + \beta E_t V(x_{t+1}, p_t).$$

If $p_{t-1} < p^N(x_t)$ or $p_t^C < p_{t-1}$ then $p_t = p_t^C$, where

$$p_t^C = \arg \max_{p_t} (\pi(x_t, p_t) + \beta E_t V(x_{t+1}, p_t)),$$

and

$$V(x_t, p_{t-1}) = \max_{p_t} (\pi(x_t, p_t) + \beta E_t V(x_{t+1}, p_t)).$$

To turn the hypothesized strategy into a subgame perfect equilibrium one may assume that any deviation from the prescribed course of action by anyone would make all firms to play myopic Nash (the worst equilibrium) forever

onwards. Subgame perfection entails the following incentive compatibility constraints:

$$V(x_t, p_{t-1}) > \max_{p_i} (\pi(x_t, p_i, p_{-i,t}) + \beta E_t V^N(x_{t+1})),$$

i.e. no firm can win by deviating from cooperation now, assuming that deviation would result in Nash-behavior from $t + 1$ on.

By making specific assumptions about the process generating x_t , and on the profit function it may be possible to prove the existence of this sort of subgame equilibrium. However, even if it were done, there would be no reason to believe in the argument as an "equilibrium selection device", since the disturbing variety of other equilibria would still exist. Therefore, we turn to agent-based modelling as an alternative approach.

4 Oligopoly Pricing by Boundedly Rational Agents

4.1 The Agent-based Model

Time consists of periods ($t = 0, 1, \dots$). At the beginning of any period each of N firms sets a single price (p_{it}) for their own differentiated good. The vector of prices is denoted by p_t . Firms have constant marginal and average costs (c_{it}), which are stochastic. The vector of marginal costs is denoted by c_t . Below we analyze several specifications for the marginal cost processes. Market demand is characterized by a logit demand system (see Anderson-de Palma 1992)

$$D_{it} = K \frac{\exp(-\frac{p_{it}}{v})}{\sum_{j=1}^N \exp(-\frac{p_{jt}}{v}) + \exp(\frac{V}{v})}.$$

Here K is the number of potential buyers, i.e the size of the market. The parameter $v > 0$ can be interpreted as the degree of differentiation. The role of $\exp(\frac{V}{v}) (> 0)$ is to make the optimal collusive price finite. Otherwise total demand would be equal to K whatever p_t is. To save space in the description below we set $K = 1$, and $v = 1$, but in the sensitivity exercises we will treat the general case.

These data define in every period a Bertrand-Chamberlin oligopoly game with constant marginal cost functions, and logit demand. Then there exists a mapping $p^0(c_t)$ determining the static Nash-equilibrium of the one-period oligopoly game. If costs are common ($c_{it} = c_{jt}$) for all pairs $i, j \in \{1, \dots, N\}$, and each t , then Anderson-de Palma (1992) established the following relationship:

$$p_t^0 = c_t + \frac{1}{1 - \frac{1}{N + \exp(p_t^0 + V)}}.$$

Though there does not exist an explicit solution, this equation is easy to solve numerically.

On the other hand one can derive the following relationship for the optimal (static) collusive price ($p^*(c_t)$):

$$N \exp(-p_t^*) = \exp(V)(p_t^* - c_t - 1).$$

By rearranging the two formulas one gets

$$p_t^0 = c_t + 1 + \frac{1}{N - 1 + \exp(p_t^0 + V)}.$$

and

$$p_t^* = c_t + 1 + \frac{N}{\exp(p_t^* + V)}.$$

Then it can be seen that for $N = 1$ (monopoly), the Nash-equilibrium and collusive prices are the same, as expected. In addition these formulas exhibit that as N increases the noncooperative price monotonically decreases, whereas the collusive price monotonically increases.

This model would be suitable in a non-inflationary environment. It is plausible to assume that demand in a partial market depends on prices relative to the general price level. As we model a partial market inflation should be treated as an exogenous variable, like marginal costs. The simplest assumption makes marginal costs and the general price level proportional, i.e.

$$P_t = \kappa c_t,$$

where P_t is the general price level, and $\kappa > 0$ is a constant. Thus we have to reinterpret all prices as nominal prices divided by the general price level. The modified formulas are as follows

$$D_{it} = \frac{\exp(-\frac{p_{it}}{P_t})}{\sum_{j=1}^N \exp(-\frac{p_{jt}}{P_t}) + \exp(V)}.$$

$$p_t^0 = c_t + P_t + \frac{P_t}{N - 1 + \exp(\frac{p_t^0}{P_t} + V)}.$$

and

$$p_t^* = c_t + P_t + \frac{NP_t}{\exp(\frac{p_t^*}{P_t} + V)},$$

The non-cooperative and collusive prices will be important benchmarks, but we will never assume that firms can calculate and apply them.

We can also allow the presence of idiosyncratic marginal costs in the model. The simplest assumption is that marginal costs are proportional to the general price level and in every period a random noise (with normal distribution and zero mean) is added to them. Relaxing the common marginal cost assumption does not change the qualitative results of the model. In the following we will present the results of the model with common marginal costs.

The firms in our model are boundedly rational agents that do not pretend to find out what an optimal pricing strategy would be. Optimality would heavily depend on the reaction of competitors, as well as on the uncertain marginal costs process. They certainly know their own costs, and have a general worldview based on the reasoning developed in the previous subsection.

Thus they know that they can "mark up" costs, in other words in a differentiated oligopoly setting prices can be profitably set above marginal cost, even if competitors would remorselessly maximize profits. They know also that there is a remote possibility to collude, and by forming a powerful cartel prices can be set "high enough" above marginal costs to achieve maximum profits for the whole industry. Furthermore, they know that there is an upper limit, even the omniscient cartel would not set infinitely high prices.

On the other hand they are sceptical, and are aware of strategic uncertainty, i.e. they are uncertain about how their competitors behave and react. This reasoning leads to these firms to formulate pricing strategies in the following way.

1. Start with a target markup range, where the lowest markup is higher than 1.
2. Observe the marginal cost and calculate the target price range.
3. Check whether your latest price falls into the target range or not.
4. If yes, do not change the price.
5. If no, set the new price as a weighted average of your latest price and the middle of the target range.
6. Observe how strategies work in the market over several periods. (Any strategy consists of the target markup range (two parameters) and the weight of the old price.) Then try to imitate successful strategies, or maybe experiment with a new one.

We note that in a previous version of the model step 5, the choice of the new price was defined as a binary decision. In that case firms set the new price at one of the extremes of the target range. That model provided the same qualitative results as the new version. However, the current version shows that price rigidity does not emerge because of the binary specification.

The change of strategies is based on a specific realization of the genetic algorithm, a learning mechanism that has been applied in several contexts within, and, mostly, without economics. In the next subsection to this we will turn.

4.2 The Learning Algorithm

The evolutionary algorithm used in this study has certain specific features. We call it evolutionary, because in spirit it is very similar to the class of algorithms surveyed for instance in Arifovic (2000).

The building of an evolutionary algorithm must start with the definition of strategies. In our case we limit the strategy space to three dimensions:

1. Setting the lower limit of the markup (a real positive number)
2. Setting the upper limit of the markup (a real positive number)

In fact it is more convenient to work with a transformation of the two real variables: setting a markup target, and a percentage deviation from the markup target.

3. Setting the weight of the latest price in the calculation of the new price when it is not between the lower and upper bounds for the markup (a real number between 0 and 1)

If the prevailing price is between the lower and the upper bounds the price is not changed.

It must be emphasized that this strategy set is obviously "much smaller", than the set of all feasible strategies. It is also more restricted than the set of available Markov-strategies. Still, these are not overly simplistic strategies, either. As we have argued in the previous subsection due reflection can lead someone to opt for these strategies in an oligopolistic situation with strategic uncertainty, thus they are far from being naive.

We start with giving more or less reasonable initial values to the 3 strategic variables. In the initial period firms act according to these (almost) random strategies, and then profits are realized.

In period 2 the strategy of each firm possibly undergoes changes, according to genetic operations. The first operation is selection or reproduction. Selection requires a measure of fitness. Suppose for now that after period 1 firms can calculate the fitness of all strategies (not only their own), having a measure of fitness for each strategy. Then we define the survival probability of the k th strategy by the Boltzmann-selection criterion as

$$p_{kt} = \frac{\exp(\frac{f_{kt}}{H})}{\sum_{j=1}^I \exp(\frac{f_{jt}}{H})},$$

where I is the number of firms, f_{kt} is the fitness measure (see on it more later) of strategy k in period t , and H is the Boltzmann-constant, frequently referred to as "temperature". At high temperatures the selection pressure is low, whereas at low temperatures it is high (i.e "only the best can survive"). Notice, that as it was mentioned survival depends on the relative fitness of a strategy

in the population, thus we model a sort of social learning process. For this purpose, we use the fitness-proportionate method, also referred to as roulette wheel sampling. This may result in some strategies being selected several times while others being dropped.

Thus chance, according to p_{kt} , determines if a strategy survives or not. Then the surviving strategies may undergo mutation: each of the three elements can be changed randomly. All variables must be given a transition probability, called the mutation rate. In addition, we specify a continuous probability distribution for the mutation. We assume that it is normal, with mean 0, and variance σ^2 . Thus here we must only specify the variance. All mutation parameters are exogenous, they do not change over time.

After disposing of the problem of strategies surviving selection one has to deal with firms whose previous strategy was dropped after the selection process. Our approach is to form a convex combination of the old (dropped) strategy, and one of those that have been selected repeatedly. This operation is not customary in the literature, one can interpret it as giving a certain individual and conservative flavor to the learning process. The weight of the dropped strategy is an exogenous parameter of our model.

Now we have to return to the fitness function. An obvious candidate is actual profits, however, intuitively measuring fitness by a single run of profits would be unreasonable. Thus we define fitness as the average of profits obtained by a strategy with exponentially declining weights.

5 The Baseline Scenario

In the following section, we briefly describe the parameters of the model and report their baseline value. Then we describe the results of this benchmark model which is followed by a sensitivity analysis in which the effect of changes in individual parameter values is tested.

5.1 Baseline Parameters

The model has several parameters, some already described in the previous sections:

1. **Number of firms:** In the baseline model there are 10 competing firms.
2. **Degree of product differentiation(v):** 1 (arbitrary)
3. **Scale of demand(K):** 1 (arbitrary)
4. V : This is the component of the logit demand system defined above which is intended to make prices finite. If it is set too low, total demand will be close to K and collusion will be excessively lucrative. If its value is too high, pricing will have no effect as the demand of each firm will almost be equal. In the baseline model its value is $-1.5c_1$.

5. **Boltzmann-constant(H):** In the baseline model H is set to 1.
6. **Fitness function:** The weights used to calculate the fitness value of a strategy stem from a geometric series. In the baseline model its quotient is 0.5.
7. **Mutation parameters:** The mutation rate is 5% for all the variables. The standard deviation of the mutation is 0.01 for the markup target, 0.02 for the weight of the old strategy and the deviation from the markup target.
8. **Weight of the repeatedly selected strategy:** When a strategy is dropped, it is replaced by the convex combination of itself and a repeatedly selected strategy. The baseline weight of the latter is 0.55.
9. **Inflation environment:** The overall annual inflation rate is a normally distributed random variable with an expected value of 3% and a 1 percentage point standard deviation.

5.2 Baseline Results

An important expectation for the model was that most of the chosen prices fall into the theoretical region, between p^N and p^{CFB} . Using the parametrization described above during the 100 test runs all prices satisfied this condition. With the baseline parameters the average duration of prices is 6.39 months, and its standard deviation is 2.88.

Another test of the model is to compare its results to actual statistics. We were able to do this by using the actual inflation data of the 3 different regions described above as inputs:

Table 1. Inflation data and percentage of prices within the theoretical region

	Monthly inflation(%)	St. dev. of inflation	Percentage of prices within the theoretical region
Hungary	0.54	0.69	98.2
USA	0.27	0.36	94.7
Eurozone	0.16	0.20	99.8

The sources of these statistics are Bauer (2008, p.255) for the Hungarian data, Klenow-Kryvstov (2008, Table 6) for the US data and Dhyne et al. (2005, p.14) for the European data.

As it can be seen in Table 2 which represents the average results of 100 runs, the model's estimates are realistic. The actual statistics themselves are only broad averages of substantially different sectorial data.

Table 2. Comparison of model results and actual statistics

	Percentage of prices changed		Average duration	
	Fact	Model's estimate	Fact	Model's estimate
Hungary	24.7	29.8	3.8	3.7
USA	36.1	19.8	6.8	5.7
Eurozone	15.3	12.7	13.0	9.5

Moreover, the baseline model behaves realistically in the asymptotic cases of inflation as well. Firstly, when the inflation rate is 0, total price rigidity arises, i.e. after in average 255 periods, none of the firms change their price for 5 years. On the other hand, we tested the algorithm in a hyperinflation environment, using the Hungarian inflation data of July 1946. The model predicted that in such an environment firms change prices in every period, as expected.

We also note that in our model no price wars emerge (i.e. there is no quick, collective fall of prices). It is true even in an artificial setting of initial strategies in which initial prices are near the collusive price p^{CFB} .

6 Sensitivity Analysis

Table 3. Sensitivity analysis - Number of firms

Number of firms	Average duration	Standard deviation
5	6.40	3.27
50	6.48	1.39
100	6.10	1.19

Somewhat surprisingly increasing the number of firms causes less variability, but average rigidity is left unaffected.

Table 4. Sensitivity analysis - Scale of demand

Scale of demand	Average duration	Standard deviation
0.5 times baseline	6.54	2.92
10 times baseline	5.63	2.18
100 times baseline	3.39	0.80

Large markets invoke less rigidity, i.e. less strategic uncertainty, which is in accordance with our intuition.

Table 5. Sensitivity analysis - Differentiation

Differentiation	Average duration	Standard deviation
0.1 times baseline	11.07	6.22
0.5 times baseline	7.25	3.08
10 times baseline	6.13	2.84

In oligopolistic models less differentiation means more intense competition. In this agent-based model with strategic uncertainty this translates into more hesitation, and more price rigidity.

Table 6. Sensitivity analysis - Temperature

Temperature	Duration	Standard deviation
0.5	6.99	2.98
2	6.61	3.93
5	6.44	2.65

Low temperature is equivalent to high selection pressure. Apparently selection pressure affects price rigidity only marginally.

Table 7. Sensitivity analysis - Fitness weights

Fitness weights	Average duration	Standard deviation
1/3	6.07	2.18
1/10	6.29	2.72
1/100	6.11	2.21

The degree of "backward looking" seems not highly relevant, which is good, since this is something we can only experiment with.

Table 8. Sensitivity analysis - Mutation rates

Mutation rates	Average duration	Standard deviation
0.01	3.31	1.09
0.1	7.62	2.97
0.2	10.07	4.91

Increasing the mutation rates clearly raises uncertainty, and, especially, strategic uncertainty. Our interpretation of the model is confirmed by finding that higher strategic uncertainty is accompanied with more price rigidity.

7 Conclusions

Our attempt to explain price rigidity by strategic uncertainty of boundedly rational agents is new in the literature. The oligopoly model in which we tried to

substantiate our claim is standard, but is far from being the simplest possible setting. The learning algorithm we proposed is founded on solid ground, but we had to adapt it to our specific problem.

It is quite promising to see that the model produces sensible results, along several dimensions. First, it reproduces the negative relationship between the level of overall inflation and price rigidity. Second, most prices fall into the theoretical "range" without explicitly building this feature into strategies. Third, price rigidity statistics are even quantitatively similar to actual data. Fourth, the sensitivity analysis show that parameter changes that intuitively correspond to higher strategic uncertainty cause indeed greater inflexibility in prices.

Obviously, this simple model cannot explain all phenomena concerning oligopolistic price setting. Its extension into several directions would test its robustness. There remains one important feature of data that the model cannot replicate: price wars. To address this issue should be our next concern.

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