

Coordination and real investments under short-sale constraints

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Abstract

This paper studies the effect of short-sale constraints on coordination among capital providers for a real investment in a rational framework. Capital providers with dispersed information consider whether or not to invest in a project, and they use an (endogenous) public signal in the form of a market price to coordinate their actions. I show that the introduction of short-sale constraints increases price volatility in an asymmetric way that, unlike in traditional coordination games, leads to multiplicity in investments and market prices too. It implies that although with no short-sale restrictions the financial market is simply a sideshow, in the presence of short-sale constraints asset prices have important feedback effects on real investments. It leads to inflated prices and undertaking negative NPV projects as well.

1 Introduction

An important strand of research in behavioural finance asks the question whether mispricings in financial markets (usually represented by irrational traders or sentiment) affect the financing and investment decisions of firms. Clearly, as Morck, Shleifer and Vishny [16] put it, "if the stock market were [only] a sideshow, market inefficiencies would merely redistribute wealth between smart investors and noise traders" and would not have feedback effects to investments at all. However, if stock prices influence real economic activity, then irrational traders can indirectly affect real activity as well.

The question how a rational manager, interested in maximizing firm value, should act when facing irrational investors has been tackled by Stein [27]. He shows that when

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a firm's stock price is too high, a rational manager should issue more shares and take advantage of the mispricing, and when the price is too low, she should repurchase shares. Regarding what to do with the fresh capital, he also shows that non-equity dependent firms should not invest straight to any investment opportunities but instead keep the money raised in cash. However, for equity-dependent firms, market mispricing could matter and could also distort investment decisions. If, for example, a firm has to raise capital for new investments in an underpriced market, they may have to forgo attractive opportunities because it is too costly to finance them with undervalued equity. On the flip side, if a rational manager refuses to undertake projects irrational investors perceive as profitable but actually they are not, they may depress stock prices or have him fired. The above stories lead to the prediction that investments of equity-dependent firms should positively correlate with stock mispricings.

This paper provides a different approach to capture the correlation between 'mispricing' and investment by studying the effect of short-sale constraints on coordination among capital providers for a real investment. Capital providers with dispersed information consider whether or not to invest in a project, and they use an (endogenous) public signal in the form of a market price to coordinate their actions. The market price is informative about the quality of the project as it reflects the information of speculators who are endowed with private signals about this project. I examine the effect of short-sale constraints (in the market) on investments and market prices. The model consists of two parallel games: the market, where speculators operate, is a noisy REE model with or without short-sale constraints. The second phase is an investment round, where capital providers consider investing in a project. The project is undertaken if the aggregate investment is sufficiently large, and investors' payoff is higher if the project is successful. It is in fact a coordination game, in the spirit of standard global games, with private and public information. Finally, the investment size affects the final payoff of the risky asset traded in the market.

In a simple financial market model, the introduction of short-sale constraints increases the price volatility as they increase potential uninformed traders' perceived uncertainty about the asset payoff by decreasing the information content of the market price. This increased price volatility means weaker information content of the public signal of investors. Unlike standard global games with public and private information, this model shows that multiplicity in the investment size and outcome prevails under short-sale constraints as the important feature is not the increased volatility but the fact that volatility increases asymmetrically. Multiplicity in investment leads to multiplicity in the price too, when investment has a feedback effect on the asset dividend. Therefore short-sale constraints might create large price jumps or falls not simply as a result of an increase in perceived uncertainty but as a result of affecting coordination.

The backbone of the present paper is a standard coordination game in the spirit of Morris and Shin [17], and builds in particular on the strand studying the interaction between private and public information (see, for example, Morris and Shin [18], [19], [20] and [21]). These models show that Atkeson [4] discusses the potential role of financial markets as the sources of endogenous public information, formalized by Angeletos and Werning [3]. They incorporate a standard Grossman and Stiglitz [11] financial market where agents with dispersed information interact, and use the rational expectations equilibrium price as public information. They show that a unique equilibrium might not emerge as a small perturbation from perfect information, due to the endogenous nature of the public signal. This paper builds on their analysis in treating the market price as a public signal, but instead focuses on the effect of introducing portfolio constraints on market participants. Therefore, the main difference is that multiplicity does not arise because the public information gets more precise when improving the quality of private information, but only because short-sale constraints confuse investors, who are not sure to what extent these constraints bind. Indeed, in the Angeletos and Werning [3] sense, the market price is exogenous, and absent any constraints, the present model simplifies to that of Morris and Shin [19] and [21].

Short-sale constraints have been associated with inflated prices or bubbles in the existing literature (see for example Miller [15], Harrison and Kreps [12] and Scheinkman and Xiong [26]; and Rubinstein [25] for many more 'anomaly' associated with short-sale constraints). These models assume that overconfident agents with heterogenous beliefs interact in markets. They show that if short-sale constraints prevent pessimists to participate, asset prices might only reflect the belief of optimists, and can even be higher than the valuation of the most optimistic market participants. Panageas [23] extends this argument to study firms' investment decisions when they raise capital in overvalued markets. Compared to these models, the present paper provides overpricing and overinvestments in a rational expectations framework. In particular, the introduction of short-sale constraints does not create overpricing in the market on its own, following the insights of Diamond and Verrecchia [8] and Bai, Chang and Wang [5]. In fact, in a fully rational framework, optimist realize that short-sale constraints rule out pessimists from the market and revise their beliefs accordingly. However, short-sale constraints increase perceived uncertainty about the asset payoff by decreasing the information content of the market price when the supply shocks are low and hence increase volatility in an asymmetric way. It is this volatility channel that affects the coordination problem of capital providers who invest even in negative NPV projects when asset prices are above fundamentals.

Yuan [28] studies a fully rational economy with asymmetric information and borrowing constraints, and shows that uninformed investors might be confused when trying to infer

the signal of informed investors. Indeed, uninformed investor demand can increase in the price: a higher price can reduce uncertainty about the constraint status of informed investors. Thus, the information effect can dominate the substitution effect of Grossman and Stiglitz, leading to a backward bending demand curve.

Feedback effects: Ozdenoren and Yuan [22]; Goldstein, Ozdenoren and Yuan [10]; Angeletos, Lorenzoni and Pavan [2]; Albagli, Hellwig and Tsyvinski [1]; Peress [24]

Empirical papers on feedback and short-sale constraints: Jones and Lamont; Chen, Goldstein and Jiang, e tc.

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The remainder of the article is organized as follows. Section 2 introduces the basic model. Section 3 studies the equilibrium of the financial market and examines the effect of short-sale constraints. Section 4 presents the investment round equilibrium and studies feedback. Finally, section 5 concludes.

2 Model

This section introduces the baseline model. I consider a three-period economy with dates $t = 0, 1$ and 2 . It consists of a round of securities trading, at date 0 , and a round of real investment, at date 1 , with information flowing from the former to the latter. Finally, assets pay off at date 2 . The economy is populated by three types of agents: speculators and noise traders are present in the financial market, while capital providers coordinate in order to undertake a corporate investment.

2.1 Securities market

2.1.1 Assets

From the viewpoint of the securities market participants, the economy is a static game with a single trading round and a final payoff. There are two securities traded in a competitive market, a risk-free bond and a risky stock. The bond is in perfectly elastic supply and is used as numeraire, with the risk-free rate normalized to 0 . The risky asset is assumed to be in random supply u , and has a random payoff V that is the sum of two random components: $V = F + v$, at date 2 . The price of the stock at date 0 is denoted by P .

2.1.2 Traders

I assume that the asset market is populated by a continuum of speculators (or informed traders) with measure one and indexed by $k \in [0, 1]$. Informed traders are risk averse and, for simplicity, I assume that they have Markowitz-Tobin mean-variance preference over terminal wealth. If W_k denotes the terminal wealth of agent k , her preference is

$$E [W_k | \mathcal{I}_k] - \frac{1}{2} \text{Var} [W_k | \mathcal{I}_k], \quad k \in [0, 1],$$

where the final wealth $W_k = x_k (V - P)$ is given by the number of shares purchased, x_k , multiplied by the profit per share, $V - P$, \mathcal{I}_k is the information set of trader k , and $E [\cdot | \mathcal{I}_k]$ and $\text{Var} [\cdot | \mathcal{I}_k]$ denote the expectation and variance conditional on \mathcal{I}_k , respectively. Before trading, informed traders observe F but not v . Also, all agents of the model (including capital providers) observe the market price P .¹ These assumptions imply that the risk associated with v will be unlearnable for everyone, thus agents with imperfect information (i.e. capital providers) will try to best guess only component F . They also impede informed traders to submit price-inelastic demand curves and

Further, I assume that informed traders might be subject to short-sale constraints. In particular, short-sale constraints mean that speculator k 's stock position is bounded below by zero: $x_k \geq 0$. Short-sale constraints can be thought of an extreme case of facing infinite costs when selling short, and the results can be easily generalized to allow for limited shorting with finite costs. I assume that a w^c proportion of informed traders are subject to short-sale constraints while the remaining, with mass $w^{uc} = 1 - w^c$, are unconstrained.

For tractability, I assume that F , v and u are uniformly and independently distributed on the intervals $[-a_F, a_F]$, $[-a_v, a_v]$ and $[-a_u, a_u]$, respectively. Thus, their variances will be $\sigma_F^2 = \frac{1}{3}a_F^2$, $\sigma_v^2 = \frac{1}{3}a_v^2$ and $\sigma_u^2 = \frac{1}{3}a_u^2$, respectively. For simplicity, I will also set $a_F = \infty$, which denotes an improper prior for F on the real line, and implies that the information set for informed traders will simply be $\mathcal{I} = \{F, P\} = \{F\}$ as the price does not contain more information about the final payoff than their information.

¹Given that in this very parsimonious model all traders are endowed with the same information and they are not heterogenous in their trading needs, the role of the random supply u is simply to generate trading. Note that in case of incorporating informational heterogeneity in the form of uninformed traders, the need for random supply u would remain in place so that the no-trade theorem of Milgrom and Stokey [14] will not apply.

2.2 Real investment

2.2.1 Capital providers and the project

After the financial trading, at $t = 1$, capital providers consider investing in a project. Capital providers (or investors, or entrepreneurs) are a continuum of agents with measure one, and indexed by $j \in [0, 1]$. Each entrepreneur can choose between two actions, either contribute one unit of capital to the project (i.e. invest): $i_j = 1$, or not $i_j = 0$. The payoff from not investing is normalized to zero. The payoff from investing is $1 - c$ if the project is undertaken and $-c$ otherwise, where $c \in (0, 1)$ parametrizes the private cost of investing. The project, in turn, is undertaken if and only if $I > \theta$, where I denotes the total capital entrepreneurs provide for the project and θ is the exogenous fundamental representing the capital need of the project.

It follows that the payoff of investor i is

$$U(i_j, I, \theta) = i_j (1_{\{I > \theta\}} - c),$$

where $1_{\{I > \theta\}}$ is the indicator of the project realization and takes the value of 1 if $I > \theta$ and 0 otherwise.

The key property of this payoff structure is the coordination motive: $U(1, I, \theta) - U(0, I, \theta)$ increases with I , so the incentive to invest increases with the mass of entrepreneurs investing, that is, investment is a strategic complement. If θ was commonly observable by all capital providers, both $I = 0$ and $I = 1$ would be an equilibrium whenever $\theta \in (0, 1]$.² In this interval the investment outcome depends on the size of the attack.

Following standard global game setups, information is assumed imperfect and asymmetric, so that θ is not common knowledge. In the beginning of the game, nature draws θ from a given distribution, which constitutes the agents' common prior about θ . For simplicity, this prior is taken to be the improper uniform over the real line. Investor j then receives a private signal $t_j = \theta + \xi_j$, where ξ_j is uniformly distributed on the interval $[-a_t, a_t]$, ξ_j is independent of θ , and independently and identically distributed across investors. Capital providers also observe a public signal, namely the price P realized in the financial market at date 1. For tractability, I will assume $\sigma_t^2 \leq \sigma_v^4 \sigma_u^2 / 4$, that is the private signal is sufficiently precise compared to the price, which eventually happens when taking the limit $\sigma_t \rightarrow 0$.

To connect the trading and the investment rounds, I will assume that the payoff of the asset, F , is related to the capital need of the project and hence the price of the asset, P , can provide additional (public) information regarding the cost of the project beyond

²See Angeletos and Werning [3].

the private signals, and hence can help (or hurt) coordination among capital providers. For simplicity, in the basic model I will assume $F = \theta$, and think about the security as a derivative, whose price is exogenously given to capital providers. In section 4.3 I will elaborate on this and assume that the payoff of the asset is increasing in the total amount invested, in particular $F = I(\theta, P)$. The endogenous payoff of the asset will create a feedback from the investment round to trading as market participants will have to forecast the collective action of capital providers.

2.3 Equilibrium

I define an equilibrium of the above game as follows.

Definition 1 *An equilibrium consists of a price function $P(F, u)$, individual strategies for trading (both constrained and unconstrained) and investing, $x_k^c(F, P)$, $x_k^{uc}(F, P)$ and $i(t_j, P)$, and their corresponding aggregates, $X(F, P)$ and $I(\theta, P)$, such that*

1. *demand is optimal for informed trader k :*

$$x_k^{uc}(F, P) \in \arg \max_{x \in \mathbb{R}} E[W_k | \mathcal{I}_k] - \frac{1}{2} \text{Var}[W_k | \mathcal{I}_k] \text{ for } k \in [0, 1], \quad (1)$$

if trader k is allowed to short, and

$$x_k^c(F, P) \in \arg \max_{x \in \mathbb{R}^+} E[W_k | \mathcal{I}_k] - \frac{1}{2} \text{Var}[W_k | \mathcal{I}_k] \text{ for } i \in [0, 1], \quad (2)$$

if shorting is prohibited for trader k ;

2. *aggregate demand and market clearing:*

$$X(s, P) = \int_0^1 x_k(s, P) dk = u \quad (3)$$

3. *investment is optimal for entrepreneur j :*

$$i(t_j, P) \in \arg \max_{i \in \{0, 1\}} E[U(i, I(\theta, P), \theta) | t_j, P], \quad (4)$$

4. *aggregate investment:*

$$I(\theta, P) = \int_0^1 i(t_j, P) dj. \quad (5)$$

Conditions (1)-(3) define a noisy rational expectations equilibrium for the trading round. In particular, condition (1) states that individual asset demands are optimal for traders with no restriction on shorting, conditioned on all available information available to informed traders, including the noisy signal and anything inferable from the price, while

(3) imposes that the asset market clears. Similarly, condition (2) states that individual asset demands are optimal for traders subject to short-sale constraints, conditioned on the noisy signal and anything inferable from the price.

Conditions (4) and (5) define a perfect Bayesian equilibrium for the investment round. I restrict my attention to monotone equilibria, defined as perfect Bayesian equilibria such that, for a given realization P of the public signal, an entrepreneur invests if and only if the realization of her private signal is lower than some threshold $t^*(P)$; that is $i(t_j, P) = 1$ iff $t_j \leq t^*(P)$. It implies that projects can be characterized in a similar way: a project with capital need θ will be undertaken if and only if this cost is lower than some threshold $\theta^*(P)$; formally if $\theta \leq \theta^*(P)$.

3 Equilibrium in the financial market

This section studies the equilibrium of the date 0 trading round before advancing to the investment round. It provides the results regarding the information content of the price.

Given the optimization problems (1) and (2), the following statement is immediate:

Proposition 2 *Speculator k 's optimal stock holding is*

$$x_k^{uc} = \frac{F - P}{\sigma_v^2} \quad (6)$$

if she is unconstrained, and

$$x_k^c = \max \left\{ \frac{F - P}{\sigma_v^2}, 0 \right\} \quad (7)$$

if she is subject to short-sale constraints.

Substituting the optimal demands 6 and 7 into the market clearing condition 3 we have

$$u = \int_0^1 x_k(s, P) dk = (w^c 1_{F > P} + w^{uc}) \frac{F - P}{\sigma_v^2},$$

and hence the market clearing price is given by

$$P = F - \frac{\sigma_v^2}{w^c 1_{u > 0} + w^{uc}} u = F - \begin{cases} au & \text{if } u > 0 \\ bu & \text{if } u \leq 0 \end{cases}, \quad (8)$$

with the notations $a \equiv \sigma_v^2$ and $b \equiv \sigma_v^2/w^{uc}$, and with $a \leq b$.

Figure 1 illustrates the main result of this section. The graph shows the asymmetric change in price contours due to the presence of short-sale constraints. When shorting is allowed (left panel), the slope of iso-price lines is the same for every realization of

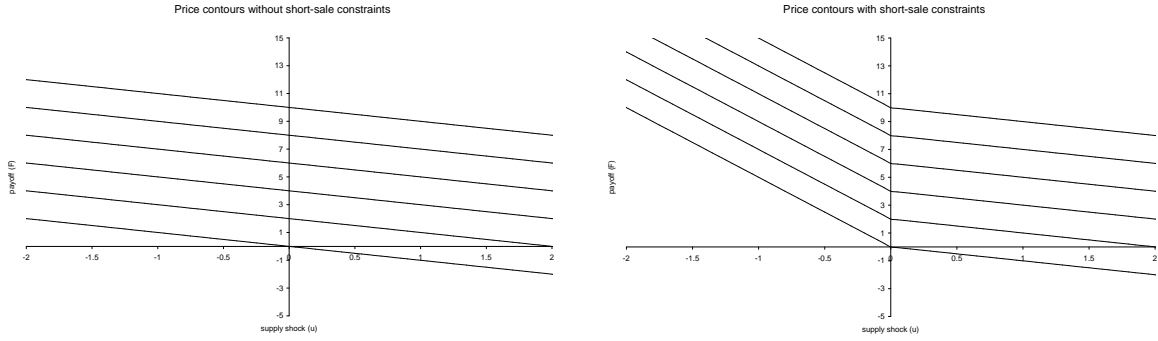


Figure 1: **State space and price contours in absence and presence of short-sale constraints.** Both graphs have the supply shock (u) on the x axis and the payoff (F) on the y axis. The left panel shows iso-price lines when investors are not short-sale constrained. The right panel shows iso-price lines when some investors are subject to short-sale constraints. The area right to the y axis is the region where the constraint does not bind and the area left to the y axis is the region where the constraint binds. The parameters are: $\sigma_v^2 = 1$, $w^c = 0.8$ and the price values are $P = 0, 2, 4, 6, 8$ and 10 .

the supply shock. When shorting is prohibited (right panel), contour lines are steeper for negative supply shocks than for positive supply shocks. It means that when the constraint binds for some speculators, a small positive change in the supply shock has a larger downward price impact, and the price reveals information about the payoff at different rates in the two regions.

3.1 The direct impact of short-sale constraints on asset prices

In order to determine the direct effect of introducing short-sale constraints in a market, we can compare conditional and unconditional first and second moments of payoff component F . In absence of short-sale constraints we have $w^{uc} = 1$ that implies $a = b = \sigma_v^2$, and the market clearing price given in equation 8 simplifies to

$$P^* = F - \sigma_v^2 u. \quad (9)$$

In absence of short-sale constraints, Bayes' rule and equation 9 imply that the distribution of payoff F conditional on price P^* is also uniform:

$$f(F|P^*) = \frac{1}{2\sigma_v^2 a_u} 1_{F \in [P^* - \sigma_v^2 a_u, P^* + \sigma_v^2 a_u]} \quad (10)$$

and hence the first two conditional moments are given by

$$E[F|P^*] = P^* \text{ and } Var[F|P^*] = \frac{1}{3} (\sigma_v^2 a_u)^2 = \sigma_v^4 \sigma_u^2. \quad (11)$$

In the presence of short-sale constraints, the conditional distribution of F is a bit more elaborate because of the different impact of supply noise on the negative or positive sides of the true value, that is when the constraint binds or not. Again, Bayes' rule combined with equation 8 implies that the conditional distribution of F is given by

$$f(F|P) = \frac{1}{2a_u} \left(\frac{1}{b} 1_{F \in [P-ba_u, P]} + \frac{1}{a} 1_{F \in [P, P+aa_u]} \right), \quad (12)$$

and hence the first two conditional moments are

$$E[F|P] = P - \frac{1}{4} \frac{1-w^{uc}}{w^{uc}} \sigma_v^2 a_u \quad \text{and} \quad Var[F|P] = \frac{5(w^{uc})^2 + 6w^{uc} + 5}{16(w^{uc})^2} \sigma_v^4 \sigma_u^2. \quad (13)$$

It is interesting to see the implications of short-sale constraints by comparing 11 and 13. If $w^{uc} < 1$, we have both $E[F|P] < E[F|P^*]$ and $Var[F|P] > Var[F|P^*]$, that is short-sale constraints decrease the conditional mean while increasing the conditional variance. Both results can best be interpreted from the viewpoint of potential uninformed traders who could enter the market. The first result, namely that the conditional mean is lower in the presence of short-sale constraints, is only the outcome of the very stylized model. Indeed, out of the two types inhabiting the market, informed traders know the true payoff F , while noise traders by definition do not try to infer the F value from the price, that is they do not 'think'. In case the model incorporated rational but uninformed traders, they would realize their lack of information and would demand a discount for their participation to make sure they break even on average. This discount is given by the difference $E[F|P^*] - E[F|P]$, and would imply that both in absence and presence of short-sale constraints, the price is an unbiased estimator of the payoff F , in line with previous rational expectations models about short-sale constraints, e.g. Diamond and Verrecchia [8] and Bai, Chang and Wang [5]. The discount $E[F|P^*] - E[F|P]$ is not important for what follows. As the focus of this paper is on the information content of prices for real investment, and because the introduction of uninformed traders would not change the information content of the market price while making the model less tractable, I refrain from this extension.³

Regarding the difference in conditional variances, equations 11 and 13 show that short-sale constraints increase potential uninformed traders' perceived uncertainty about the asset payoff by decreasing the information content of the market price, as in Bai, Chang and Wang [5].

³The interested reader should refer to the methodological contribution of Yuan [28] who investigates asymmetric price movements under borrowing constraints.

4 Application: Investments

At date 1, after speculators' trade in the financial market has taken place, entrepreneurs have the opportunity to invest in a project. The equilibrium price of the financial market, P or P^* , provides an observable public signal, hence investors can coordinate their actions based on it. In the following subsections I solve the coordination model, first without constraints on short-selling, then with short-sale constraints. I restrict my attention to monotone equilibria, defined as perfect Bayesian equilibria such that, for a given realization P of the public signal, an entrepreneur invests if and only if the realization of her private signal is below some threshold $t^*(P)$, that is $i(t_j, P) = 1$ iff $t_j \leq t^*(P)$; and a project with capital need θ is undertaken if and only if the cost is lower than some threshold $\theta^*(P)$; that is iff $\theta \leq \theta^*(P)$. I also assume that the capital need of the project, θ , and the payoff of the stock, V , are positively related. In what follows, for simplicity, I set $\theta = F$, which is relaxed later.

4.1 Equilibrium analysis with no short-sale constraints

In this section I provide a solution to the coordination game among capital providers when short-selling is allowed for everyone. The setup of the model ensures that the information spillover from the financial market to real investments, in the form of the price P , can be treated as exogenous to entrepreneurs. It implies that the equilibrium of the whole economy is just a simple conjugate of the equilibrium of the trading round, discussed in the previous section, and the equilibrium of the investment round, with these two being separated from each other. This subsection hence only solves for the equilibrium of a standard global game setup with private and public information, fully characterized by conditions (4) and (5).

In a monotone equilibrium described above, the aggregate size of investment is given by

$$I(\theta, P^*) = \Pr(t \leq t^*(P^*) | \theta) = \begin{cases} 0 & \text{if } t^*(P^*) < \theta - a_t \\ \frac{t^*(P^*) - (\theta - a_t)}{2a_t} & \text{if } \theta - a_t \leq t^*(P^*) < \theta + a_t \\ 1 & \text{if } \theta + a_t \leq t^*(P^*). \end{cases}$$

The project is undertaken if and only if $\theta \leq \theta^*(P^*)$, where $\theta^*(P^*)$ solves $I(\theta, P^*) = \theta$, therefore

$$\theta^*(P^*) = \begin{cases} 0 & \text{if } t^*(P) < -a_t \\ \frac{t^*(P) + a_t}{1 + 2a_t} & \text{if } -a_t \leq t^*(P) < 1 + a_t \\ 1 & \text{if } 1 + a_t \leq t^*(P). \end{cases} \quad (14)$$

It follows that the expected payoff of agent j from investing is $\Pr(\theta < \theta^*(P^*) | t_j, P^*) -$

c and hence $t^*(P^*)$ must solve the indifference condition $\Pr(\theta < \theta^*(P^*) | t_j, P^*) = c$. Posteriors about θ , conditional on t_i and P , can be computed similarly to (10). In absence of short-sale constraints, since both the public signal and the private signals are uniformly distributed, over $[\theta - \sigma_v^2 a_u, \theta + \sigma_v^2 a_u]$ and $[\theta - a_t, \theta + a_t]$, respectively, the assumption $\sigma_t^2 \leq \sigma_v^4 \sigma_u^2 / 4$ implies that investor j only uses her private signal to predict the cost⁴:

$$f(\theta | t_j, P^*) = \frac{1}{2a_t} \mathbf{1}_{\theta \in [t_j - a_t, t_j + a_t]}.$$

Thus, the indifference condition is equivalent to

$$\frac{\theta^*(P^*) - (t^*(P^*) - a_t)}{2a_t} = c,$$

that is

$$\theta^*(P^*) = t^*(P^*) - (1 - 2c) a_t. \quad (15)$$

An equilibrium is the joint solution to conditions (14) and (15), which imply

$$\theta^*(P^*) = 1 - c$$

and

$$t^*(P^*) = 1 - c + a_t(1 - 2c).$$

The following proposition states the above result:

Proposition 3 *In the game with no short-sale constraints, the equilibrium is unique if $\sigma_t \leq \sigma_v^2 \sigma_u / 2$. Moreover, as private noise vanishes so that $\sigma_t \rightarrow 0$, every investor with private signals below $t^* = 1 - c$ invest, and every project with capital need less than $\theta^* = 1 - c$ are undertaken.*

Proposition 3 states the well-known Morris-Shin result. For any positive level of noise in the public signal ($\sigma_v^2 \sigma_u > 0$), uniqueness is ensured by sufficiently small noise in the private signal. The intuition is that the more precise is private information relative to public information, the more heavily entrepreneurs use their private information. This makes it harder to predict the actions of others, heightening strategic uncertainty. When this effect is strong enough, multiplicity breaks down.

As private information becomes arbitrarily precise, $\sigma_t \rightarrow 0$, individuals stop relying on the public signal, and hence the equilibrium dependence on the common noise components (v and u) vanishes. It implies that in the limit, the investment outcome does not depend on public signal P .

⁴This is only true for $P^* - \sigma_v^2 a_u + a_t \leq t_i \leq P^* + \sigma_v^2 a_u - a_t$; for t_i being far away from the price realization, the limits must be adjusted accordingly.

4.2 Equilibrium analysis with short-sale constraints

As shown in section 3, the introduction of short-sale constraints has an adverse effect on the market price as it amplifies the effect of supply noise when the supply shock is low. The fact that the price reveals information about the payoff at different rates for low (negative) and high (positive) shocks implies that short-sale constraints notably change the inference problem of entrepreneurs who assign positive probabilities to being on these two different sides.

Given that the joint distribution of θ and the private signals does not change, the critical mass condition 14 does not change. Short-sale constraints only affect the posterior of investors after observing both the price and the private signal. As the public signal is not uniformly distributed any more even if the assumptions $\sigma_t^2 \leq \sigma_v^4 \sigma_u^2 / 4$ and $P - \sigma_v^2 a_u + a_t \leq t_j \leq P + \sigma_v^2 a_u - a_t$ still hold, the posterior will depend on the price for those who have private signals not significantly different from P . In fact, when the price is much higher than the more precise private signal of an investor, it is straightforward that she should only use the private signal for the posterior, as she is able to infer that large supply noise pushes the asset price down and thus the constraint does not bind for informed traders. Similarly, when the price is much higher than the private signal of a capital provider, she again only uses the private information for the inference. In this situation she can tell that the price is too high, which signals a very low supply shock and implies that informed investors are short-sale constrained. Given the distributional assumptions, in these two cases entrepreneurs only use the private signal; formally, we have $f_{\theta|P,t}(\theta|P, t_j) = 1/(2a_t) 1_{F \in [t_j - a_t, t_j + a_t]}$ for both $P - ba_u + a_t \leq t_j \leq P - a_t$ and $P + a_t \leq t_j \leq P + aa_u - a_t$.

In the intermediate region, when the price and the private signal realization are not far from each other, that is $|P - t_j| \leq a_t$, the investor is confused whether the short-sale constraint is binding for speculators. Using Bayes' law, the posterior distribution is given by

$$f_{\theta|P,t_j}(\theta|t_j, P) = \frac{\frac{1}{b} 1_{\theta \in [t_j - a_t, P]} + \frac{1}{a} 1_{\theta \in [P, t_j + a_t]}}{\frac{1}{b} [P - (t_j - a_t)] + \frac{1}{a} [t_j + a_t - P]}.$$

(plot, describe)

The expected payoff of agent j from investing is $\Pr(\theta < \theta^*(P) | t_j, P) - c$ and hence $\theta^*(P)$ must solve the indifference condition $\Pr(\theta < \theta^*(P) | t_j, P) = c$ which is equivalent to

$$\theta^* = \begin{cases} (1-c)(t^* - a_t) + c(t^* + a_t) & \text{if } P - ba_u + a_t \leq t^* \leq P - a_t \\ (1-c)(t^* - a_t) + \frac{b}{a}c(t^* + a_t) - c\frac{b-a}{a}P & \text{if } P - a_t \leq t^* \leq P + a_t \text{ and } \theta^* \leq \frac{P}{(16)} \\ (1-c)\frac{a}{b}(t^* - a_t) + c(t^* + a_t) + (1-c)\frac{b-a}{b}P & \text{if } P - a_t \leq t^* \leq P + a_t \text{ and } \theta^* > \frac{P}{(16)} \\ (1-c)(t^* - a_t) + c(t^* + a_t) & \text{if } P + a_t \leq t^* \leq P + aa_u - a_t \\ (1-c)\frac{b-a}{b}[(t^* - a_t) - P] & \end{cases} \quad (17)$$

Figure 2 illustrates the individual optimality condition, (16), and the critical mass condition, (14). The latter is unchanged compared to the absence of short-sale constraints. The former, however, exhibits a hump shape in the middle. When the price realization is either very high or very low, all investors know whether short-sale constraints bind for informed investors (in the former case they do, in the latter they do not) and they all put the same weight on the private signal. It implies a linear relationship with slope 1 between θ^* and t^* , which is represented for $t^* < 0.2$ and $t^* > 1.2$ on the graph: an increase of Δt in the individually optimal threshold t^* means that exactly Δt more projects will be undertaken.

For intermediate price realizations there are some agents who attach positive probabilities for both the constrained and unconstrained regions. As long as the majority of entrepreneurs obtain signals below the price $P = 0.7$, an increase of Δt in the individually optimal threshold t^* accounts for an increase of $(1-c + \frac{b}{a}c)\Delta t > \Delta t$ in the optimal investment threshold, because these new agents willing to invest will increase the weight they put on the price by $\frac{b-a}{a}c\Delta t$. Thus, θ^* grows faster than t^* . Similarly, when the majority of investors have private signals above the price, and increase in the number of investors by Δt implies that in aggregate they put $(1-c)\frac{b-a}{b}\Delta t$ less weight on the price, and θ^* grows slower than t^* . The presence of short-sale constraints thus creates a varying relative precision between the private and the public signal.

Combining conditions (14) and (16), the equilibrium is characterized by the following proposition:

Proposition 4 *If $1 + 2a_t < \frac{b}{(1-c)a+bc}$, we have*

$$t^*(P) = \begin{cases} 1 - c + a_t(1 - 2c) & \text{if } (1-c)(1 + 2a_t) - aa_u \leq P \leq 1 - c(1 + 2a_t) \\ \frac{[1+(1+2a_t)\frac{(1-c)a-bc}{b}]}{[1+(1+2a_t)\frac{(1-c)a+bc}{b}-1]}a_t - \frac{(1+2a_t)(1-c)(1-\frac{a}{b})}{[(1+2a_t)\frac{(1-c)a+bc}{b}-1]}P & \text{if } \frac{(1-c)a}{(1-c)a+bc} \leq P \leq 1 - c(1 + 2a_t) \\ \frac{[1+(1+2a_t)\frac{(1-c)a-bc}{a}]}{[(1+2a_t)\frac{(1-c)a+bc}{a}-1]}a_t - \frac{(1+2a_t)c(1-\frac{b}{a})}{[(1+2a_t)\frac{(1-c)a+bc}{a}-1]}P & \text{if } \frac{(1-c)a}{(1-c)a+bc} \leq P \leq (1-c)(1 + 2a_t) \\ 1 - c + a_t(1 - 2c) & \text{if } (1-c)(1 + 2a_t) \leq P \leq 1 - c(1 + 2a_t) + ba_u. \end{cases}$$

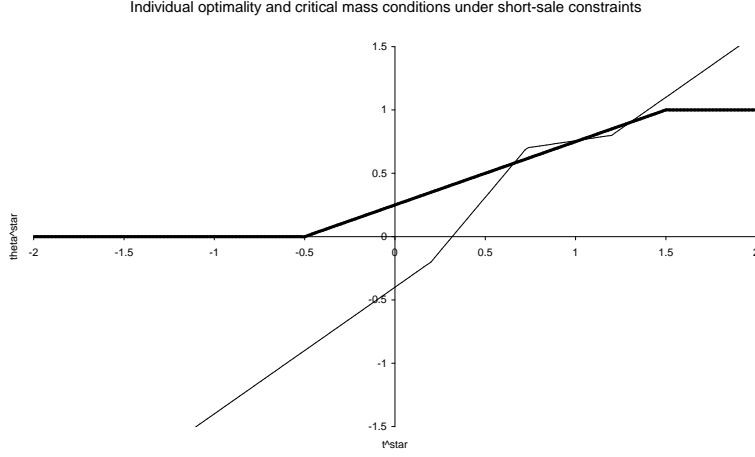


Figure 2: **Individual optimality and critical mass conditions under short-sale constraints.** This figure plots the individual optimality, (14), thin line, and the critical mass condition, (16), thick line, under short-sale constraints. The investment strategy t^* is shown on the x axis and the investment threshold θ^* is on the y axis. The parameters used here are $F = 0.1$, $u = 0.2$, $P = 0.7$, $\sigma_v^2 = 1$, $w^c = 0.875$, $a_u = 1.7$, $a_t = 0.5$ and $c = 0.1$.

and

$$\theta^*(P) = \begin{cases} 1 - c & \text{if } (1 - c)(1 + 2a_t) - aa_u \leq P \leq 1 - c(1 + 2a_t) \\ \frac{2(1-c)a}{[(1+2a_t)\frac{(1-c)a+bc}{b} - 1]} a_t - \frac{(1-c)(1-\frac{a}{b})}{[(1+2a_t)\frac{(1-c)a+bc}{b} - 1]} P & \text{if } \frac{(1-c)a}{(1-c)a+bc} \leq P \leq 1 - c(1 + 2a_t) \\ \frac{2(1-c)}{[(1+2a_t)\frac{(1-c)a+bc}{a} - 1]} a_t - \frac{c(1-\frac{b}{a})}{[(1+2a_t)\frac{(1-c)a+bc}{a} - 1]} P & \text{if } \frac{(1-c)a}{(1-c)a+bc} \leq P \leq (1 - c)(1 + 2a_t) \\ 1 - c & \text{if } (1 - c)(1 + 2a_t) \leq P \leq 1 - c(1 + 2a_t) + ba_u. \end{cases}$$

If $1 + 2a_t > \frac{b}{(1-c)a+bc}$, we have

$$t^*(P) = \begin{cases} 1 - c + a_t(1 - 2c) & \text{if } (1 - c)(1 + 2a_t) - aa_u \leq P \leq 1 - c(1 + 2a_t) \\ \frac{[1+(1+2a_t)\frac{(1-c)a-bc}{b}]}{[(1+2a_t)\frac{(1-c)a+bc}{b} - 1]} a_t - \frac{(1+2a_t)(1-c)(1-\frac{a}{b})}{[(1+2a_t)\frac{(1-c)a+bc}{b} - 1]} P & \text{if } 1 - c(1 + 2a_t) \leq P \leq \frac{(1-c)a}{(1-c)a+bc} \\ \frac{[1+(1+2a_t)\frac{(1-c)a-bc}{a}]}{[(1+2a_t)\frac{(1-c)a+cb}{a} - 1]} a_t - \frac{(1+2a_t)c(1-\frac{b}{a})}{[(1+2a_t)\frac{(1-c)a+cb}{a} - 1]} P & \text{if } \frac{(1-c)a}{(1-c)a+bc} \leq P \leq (1 - c)(1 + 2a_t) \\ 1 - c + a_t(1 - 2c) & \text{if } (1 - c)(1 + 2a_t) \leq P \leq 1 - c(1 + 2a_t) + ba_u. \end{cases}$$

and

$$\theta^*(P) = \begin{cases} 1 - c & \text{if } (1 - c)(1 + 2a_t) - aa_u \leq P \leq 1 - c(1 + 2a_t) \\ \frac{2(1-c)a}{[(1+2a_t)\frac{(1-c)a+bc}{b} - 1]} a_t - \frac{(1-c)(1-\frac{a}{b})}{[(1+2a_t)\frac{(1-c)a+bc}{b} - 1]} P & \text{if } 1 - c(1 + 2a_t) \leq P \leq \frac{(1-c)a}{(1-c)a+bc} \\ \frac{2(1-c)}{[(1+2a_t)\frac{(1-c)a+cb}{a} - 1]} a_t - \frac{c(1-\frac{b}{a})}{[(1+2a_t)\frac{(1-c)a+cb}{a} - 1]} P & \text{if } \frac{(1-c)a}{(1-c)a+bc} \leq P \leq (1 - c)(1 + 2a_t) \\ 1 - c & \text{if } (1 - c)(1 + 2a_t) \leq P \leq 1 - c(1 + 2a_t) + ba_u. \end{cases}$$

In the limit when $a_t \rightarrow 0$, the above conditions simplify to

$$t^*(P) = \begin{cases} 1 - c & \text{if } (1 - c) - aa_u \leq P \leq 1 - c \\ P & \text{if } \frac{(1-c)a}{(1-c)a+bc} \leq P \leq 1 - c \\ 1 - c & \text{if } 1 - c \leq P \leq 1 - c + ba_u. \end{cases}$$

and

$$\theta^*(P) = \begin{cases} 1 - c & \text{if } (1 - c) - aa_u \leq P \leq 1 - c \\ P & \text{if } \frac{(1-c)a}{(1-c)a+bc} \leq P \leq 1 - c \\ 1 - c & \text{if } 1 - c \leq P \leq 1 - c + ba_u, \end{cases}$$

and lead to the main result of the section:

Proposition 5 *In the presence of short-sale constraints, there are multiple equilibria for $\sigma_t^2 \leq \sigma_v^4 \sigma_u^2 / 4$ and $a_t < \frac{(1-c)(b-a)}{2[(1-c)a+bc]}$. Moreover, as private noise vanishes so that $\sigma_t \rightarrow 0$, there exist thresholds $\underline{\theta} < \bar{\theta} = 1 - c$ such that for $\theta \in [\underline{\theta}, \bar{\theta}]$ both a 'high investment' equilibrium and a 'low investment' equilibrium exist.*

The high investment equilibrium is the same as the unique equilibrium of the unconstrained economy: projects with capital need below $\bar{\theta} = 1 - c$ are undertaken, which is the socially optimal outcome. However, there exist a low investment equilibrium as well, when projects with cost between the public signal P and $1 - c$ are foregone, despite having a positive net present value and hence the socially optimal action would be to invest. This is the adverse effect of short-sale constraints on coordination.

4.3 Feedback

In this section I introduce feedback from the investment decision to the asset payoff. The aim is to show that multiple equilibria in investments can transmit multiplicity to prices, too.

The model is exactly as in the previous section, except for the endogeneity of the final payoff. In particular, I assume that informed traders receive their private information about the capital requirement of the project, θ , and that the final dividend is a function of the aggregate size of the investment. To preserve the informational structure of the model, I assume an increasing linear function of I : $F = I$, and that the technical restriction $1 < aa_u$ holds.⁵ The informed traders can be interpreted as insiders of the firm (e.g. managers) who have information regarding the potential project and trades in the market

⁵To be precise, only the restriction

$$\max\{c, 1 - c\} \frac{w^{uc}}{1 - c + cw^{uc}} \leq \sigma_v^2 a_u$$

in the hope of transmitting their (positive) information to other market participants. However, it is crucial that these insiders do not have market power, therefore they are not able to manipulate prices as in Goldstein and Guembel [9].

The derivation of the equilibrium is similar to the previous steps, and this time I only consider the general case with partially prohibited short-selling. The unconstrained case obtains with setting $w^{uc} = 1$, or $b = a$.

In monotone equilibria, a capital provider invests if and only if her private signal is below some threshold $t^*(P)$, so aggregate investment is given by

$$I(\theta, P) = \begin{cases} 0 & \text{if } t^*(P) < \theta - a_t \\ \frac{t^*(P) - (\theta - a_t)}{2a_t} & \text{if } \theta - a_t \leq t^*(P) < \theta + a_t \\ 1 & \text{if } \theta + a_t \leq t^*(P), \end{cases}$$

and hence the realized dividend is

$$F = \begin{cases} 0 & \text{if } t^*(P) < \theta - a_t \\ \frac{t^*(P) - (\theta - a_t)}{2a_t} & \text{if } \theta - a_t \leq t^*(P) < \theta + a_t \\ 1 & \text{if } \theta + a_t \leq t^*(P), \end{cases}$$

In what follows, I conjecture and later verify that $\theta - a_t \leq t^*(P) < \theta + a_t$ if a_t is sufficiently small, which will hold when taking the limit $a_t \rightarrow 0$. Speculator k 's stock demand is now given by

$$x_k^{uc} = \frac{F - P}{\sigma_v^2} = \frac{t^*(P) - (\theta - a_t) - 2a_tP}{2a_t\sigma_v^2}$$

if she is unconstrained, and

$$x_k^c = \max \left\{ \frac{t^*(P) - (\theta - a_t) - 2a_tP}{2a_t\sigma_v^2}, 0 \right\}$$

if she is subject to short-sale constraints. Thus, aggregate demand equals

$$X(\theta, P) = \left(w^c 1_{\frac{t^*(P) - (\theta - a_t) - 2a_tP}{2a_t\sigma_v^2} > 0} + w^{uc} \right) \frac{t^*(P) - (\theta - a_t) - 2a_tP}{2a_t\sigma_v^2},$$

is needed to make sure that the region for non-monotonic demand curve exists. It means that either source of non-fundamental volatility (a_u and σ_v) should be relatively high or a large proportion of informed traders should be short-sale constrained.

and market clearing implies

$$\begin{aligned}
t^*(P) + a_t - 2a_tP &= \theta + \frac{2a_t\sigma_v^2}{(w^c 1_{t^*(P)+a_t-2a_tP>\theta} + w^{uc})} u \\
&= \theta + \frac{2a_t\sigma_v^2}{(w^c 1_{t^*(P)+a_t-2a_tP>\theta} + w^{uc})} u, \\
\tilde{P} &= \theta + \frac{2a_t\sigma_v^2}{w^c 1_{\theta<\tilde{P}} + w^{uc}} u = \theta + \begin{cases} 2a_t a u & \text{if } u > 0 \\ 2a_t b u & \text{if } u \leq 0 \end{cases},
\end{aligned}$$

with the usual notations and the introduction of $\tilde{P} \equiv t^*(P) + a_t - 2a_tP$ that denotes the price of an asset with payoff $\tilde{F} = \theta = t^*(P) + a_t - 2a_tP$. I restrict my attention to equilibria with a one-to-one mapping between P and \tilde{P} so they are observationally equivalent, for which it must be that $t^*(P) - 2a_tP$ is strictly monotonic in P .

As the investment phase is identical to the benchmark model, except for the endogeneity of the payoff, the thresholds θ^* and t^* must solve versions of equations 14 and 16, but with \tilde{P} replacing P . The critical mass condition is hence unchanged:

$$\theta^*(P) = \begin{cases} 0 & \text{if } t^*(P) < -a_t \\ \frac{t^*(P)+a_t}{1+2a_t} & \text{if } -a_t \leq t^*(P) < 1+a_t \\ 1 & \text{if } 1+a_t \leq t^*(P). \end{cases} \quad (18)$$

The posterior distribution of capital providers, conditional on observing \tilde{P} and t_j , is given by

$$f_{\theta|t_j, \tilde{P}}(\theta|t_j, \tilde{P}) = \begin{cases} \frac{1}{2a_t} 1_{\theta \in [t_j - a_t, t_j + a_t]} & \text{if } \tilde{P} - 2a_t a a_u + a_t \leq t_j \leq \tilde{P} - a_t \\ \frac{\frac{1}{a} 1_{\theta \in [t_j - a_t, \tilde{P}]} + \frac{1}{b} 1_{F \in [\tilde{P}, t_j + a_t]}}{\frac{1}{a} [\tilde{P} - (t_j - a_t)] + \frac{1}{b} [t_j + a_t - \tilde{P}]} & \text{if } \tilde{P} - a_t \leq t_j \leq \tilde{P} + a_t \\ \frac{1}{2a_t} 1_{\theta \in [t_j - a_t, t_j + a_t]} & \text{if } \tilde{P} + a_t \leq t_j \leq \tilde{P} + 2a_t b a_u - a_t, \end{cases}$$

hence the individual optimality condition $\Pr(\theta < \theta^*(P) | t^*(P), P) = c$ is equivalent to

$$\theta^*(P) = \begin{cases} t^*(P) - (1 - 2c) a_t & \text{if } 1 - a a_u \leq P \leq 0 \\ t^*(P) - (1 - 2c) a_t - 2a_t c \frac{b-a}{b} P & \text{if } 0 \leq P \leq \frac{(1-c)b}{(1-c)b+ca} \\ t^*(P) - \left((1-c) \frac{2b-a}{a} - c \right) a_t + 2a_t (1-c) \frac{b-a}{a} P & \text{if } \frac{(1-c)b}{(1-c)b+ac} \leq P \leq 1 \\ t^*(P) - (1 - 2c) a_t & \text{if } 1 \leq P \leq b a_u. \end{cases} \quad (19)$$

Combining (18) and (19) yields the equilibrium thresholds

$$t^*(P) = \begin{cases} (1-c) + a_t(1-2c) & \text{if } 1 - aa_u \leq P \leq 0 \\ (1-c) + a_t(1-2c) + (1+2a_t)c\frac{b-a}{b}P & \text{if } 0 \leq P \leq \frac{(1-c)b}{(1-c)b+ca} \\ (1-c)\frac{b}{a} + a_t\left(\frac{2b-a}{a} - 2\frac{b}{a}c\right) - (1+2a_t)(1-c)\frac{b-a}{a}P & \text{if } \frac{(1-c)b}{(1-c)b+ac} \leq P \leq 1 \\ (1-c) + a_t(1-2c) & \text{if } 1 \leq P \leq ba_u \end{cases}$$

and

$$\theta^*(P) = \begin{cases} 1-c & \text{if } 1 - aa_u \leq P \leq 0 \\ 1-c + c\frac{b-a}{b}P & \text{if } 0 \leq P \leq \frac{(1-c)b}{(1-c)b+ca} \\ \frac{b}{a}(1-c) - (1-c)\frac{b-a}{a}P & \text{if } \frac{(1-c)b}{(1-c)b+ac} \leq P \leq 1 \\ 1-c & \text{if } 1 \leq P \leq ba_u. \end{cases}$$

The thresholds $t^*(P)$ and $\theta^*(P)$, and hence the demand $X(\theta, P)$ are uniquely determined, and both t^* and θ^* first increasing then decreasing functions of the price. Therefore, as the final payoff $F = (t^*(P) + a_t - \theta) / (2a_t)$ increases in P in the region $0 \leq P \leq \frac{(1-c)b}{(1-c)b+ca}$, asset demand does not necessarily decrease in P . In fact,

$$\frac{\partial X(\theta, P)}{\partial P} = \begin{cases} 0 & \text{if } 1 - aa_u \leq P \leq 0 \\ \frac{1+2a_t}{2a_t}c\frac{b-a}{b} - 1 & \text{if } 0 \leq P \leq \frac{(1-c)b}{(1-c)b+ca} \\ -\frac{1+2a_t}{2a_t}(1-c)\frac{b-a}{a} - 1 & \text{if } \frac{(1-c)b}{(1-c)b+ac} \leq P \leq 1 \\ 0 & \text{if } 1 \leq P \leq ba_u, \end{cases}$$

thus, for $\frac{1}{2}\frac{c(b-a)}{(1-c)b+ca} > a_t$, the demand is non-monotone. The following proposition states the above result:

Proposition 6 *Suppose $1 < aa_u$; and take the limit $a_t \rightarrow 0$. The demand curve is backward-bending if and only if short-selling is prohibited.*

The backward-bending demand curve is possible here because of the feedback between the financial market and real investments. A higher price realization makes entrepreneurs more inclined to invest, whereas higher investment raises the asset dividend. Provided that this informational effect is stronger than the standard Grossman-Stiglitz substitution effect, informed traders' demand can increase with its price over some region.

Solving for the equilibrium price we obtain the following proposition:

Proposition 7 *Consider the limit $a_t \rightarrow 0$. In the absence of short-sale constraints the unique equilibrium is characterized by*

$$t^*(P) = \theta^*(P) = 1 - c,$$

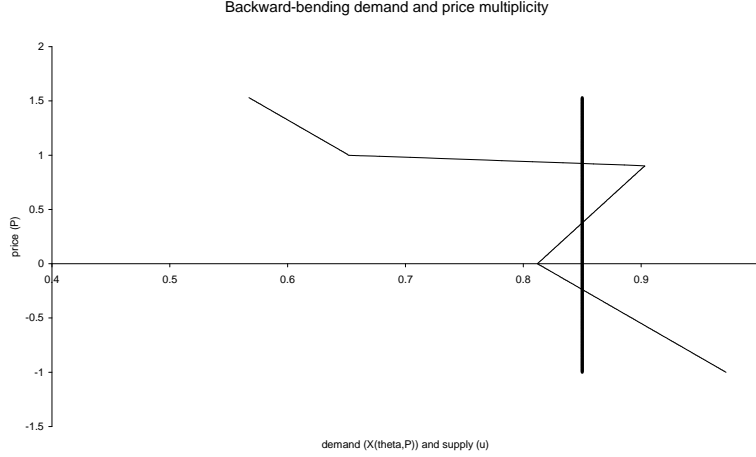


Figure 3: **Backward-bending asset demand and price multiplicity.** This graph shows that for small private signal noise the feedback from investment to asset payoff can create a non-monotonic demand curve (represented by the thin line) that intersects up to three times with the supply curve (thick line).

thus, the thresholds and hence the investment outcome are all independent of non-fundamental noise.

In the presence of short-sale constraints, there are a continuum of equilibria in both the market price and investments that support each other. In particular, the price and the thresholds are given by

$$P = \frac{a}{b-a} \left(\frac{b}{a} - \frac{\theta}{1-c} \right) = \frac{w^{uc}}{1-w^{uc}} \left(\frac{1}{w^{uc}} - \frac{\theta}{1-c} \right) \text{ and}$$

$$t^*(P) = \theta^*(P) = (1-c) \left(\frac{b}{a} - \frac{b-a}{a} P \right) = (1-c) \frac{1 - (1-w^{uc})P}{w^{uc}}$$

with $\frac{(1-c)b}{(1-c)b+ac} \leq P \leq 1$ and $1-c \leq t^*(P) = \theta^*(P) \leq \frac{(1-c)b}{(1-c)b+ac}$.

Proposition 7 describes the equilibria in the absence and presence of short-sale constraints. When shorting is not restricted, the stock market is indeed just a sideshow, and the thresholds and hence the investment outcome are all independent of non-fundamental noise represented by the market price. However, when shorting is prohibited, there exist a continuum of equilibria for the inflated prices $\frac{(1-c)b}{(1-c)b+ac} \leq P \leq 1$. In all these cases, investment decisions do depend on the market price and therefore mispricing does affect corporate investments. As $1-c \leq t^*(P) = \theta^*(P) \leq \frac{(1-c)b}{(1-c)b+ac}$, we can conclude that the model features overinvestments as even some negative NPV projects are undertaken.

5 Conclusion

To be completed.

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