Occupational mismatch and social networks*

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Abstract

We investigate the question when do the jobs found through social networks provide higher wages on average than the ones obtained on the market. In our model of heterogenous workers and heterogenous jobs the wages associated to a search method depend on the frequency that the given method provides information about jobs where the worker's productivity is high. The performance of the formal market is increased by the intensity of search to find appropriate jobs. On the other hand, the performance of social networks depends on the type of contacts an unemployed has: professional and family contacts. Professional contacts always provide good information while family members transmit good offers with higher probability if they are of the same type as the unemployed (ie. the family network is homophilous). We show that for any search intensity and number of professional contacts the family network always can be sufficiently homophilous such that the social networks give better offers on average.

1 Introduction

1.1 Motivation

Workers often use social contacts while searching for a job in addition to the usual formal methods of job search such as newspaper ads or direct application to employers (see for example, Blau and Robins (1990), Holzer (1987)). The mostly cited number says that approximately 50% of the jobs were found through informal methods. Workers being placed by contacts tend to show higher satisfaction with their job (Granovetter (1995)) and experience longer job tenure (Simon and Warner (1992), Loury (2006)). Although this would imply that worker-employer matches formed

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through social networks are better on average than those occuring through the markets, estimates of average wages do not fully confirm this idea: Simon and Warner (1992), Kugler (2003), Dustman et al. (2010) find a wage premium for network search while Bentolila (2009) finds a wage discount, Pellizziari (2009) obtains mixed evidence in different countries.

Our main question in this study is when networks perform better than the market in producing good matches. We look at the market as a random arrival process where the search by the unemployed increases the probability that the market actually provides a good offer.

On the other hand, the networks mean two types of relationships in our model which corresponds to different roles social contacts may play. One such role might be favoritism. As Bramoullé and Goyal (2009) puts it "Favoritism refers to the action of offering jobs, contracts and resources to members of one's own social group to the detriment of others outside the group". In our model this means that agents engaged in a favoring relationship and having a job offer first check out with each other whether there is a need for that offer, independently of the fact that actually the offer is not suited for the partner. In principle this practice should be a source of mismatch in the society. Bentolila et al (2009) estimates wage discount for jobs found through family contacts, hence in our model we call this relationship as family relationship. [1]

Social networks not only connect family members and friends willing to practice favoritism toward each other but professional acquaintances as well who are less likely to do such favors. This is so because they associate less personal value to the ties of this nature and as well they care more about their professional reputation toward the employers: they do not recommend someone who does not have the exact abilities to perform well in a job. Saloner (1985) shows that this reputation effect prevents the job referral to lie about unobserved abilities of the candidate. There is some empirical evidence as well that work-related contacts (previous co-workers, employers, acquaintances from professional clubs and schools)) provide better paying jobs than family members (Granovetter (1994), Bridges and Villemez (1986)).

To investigate how these two types of social networks determine the level of mismatch in the society, we build an equilibrium search model a la Pissarides where we derive the matching function using assumptions on the transmission of information by social contacts. We consider two types of workers and two types of jobs (two sectors), worker's productivity is high in one type of job and low in the other. Employers and employees accept bad matches due to market frictions and costly waiting for consecutive matches. Offers arrive at random to unemployed agents. We parameterize the search intensity of the unemployed toward their corresponding high productivity

[1] The same conclusion is drawn by Granovetter (1994) and Bridges and Villemez (1986). Sprengers (1988) and Loury (2006) shows that family is often used as last resort which also points to the direction that family might be source of mismatch.

jobs which increases the probability that the arriving job is actually a good one. On the other hand, we assume that employed agents have direct access only to the information about the vacancies of the sector of their current job.

Workers have two types of relationships: k_P of their contacts are professional ones, who forward only good matches since they do not want to introduce bad candidates to the employer (reputation effect). On the other hand, individuals have k_F family members who prefer to help the employment of the individual even if the match is not good. Their reputation considerations are overwritten by a kind of altruistic act toward their family members or by knowing that in reverse case they will be favored as well (Bramoullé and Goyal (2009)). Note that in our model the contact type (professional or family) is in reality the characteristics of the link, not the node: every agent first checks whether among family members there is a need for the offer, and if not, she looks around among professional contacts but only transmits offers which result in good matches.

Further, ties may connect similar or different types of workers. We assume that family ties connect same type of agents with probability γ and different type of agents with probability $1 - \gamma$ where γ is a measure of homophily inside the family. On the other hand, we suppose that professional ties always connect agents of similar type. This reflects the idea that these ties are often formed in meetings, schools where agents of the same occupation meet or with the intention of having access to information of one's own profession.

Our main results show that essentially the homophily parameter is decisive regarding the relationship between the expected wages provided by the market and the social networks. First, we investigate the case when there are no professional contacts in the model. In this case, we show that the expected wage of the market arrival is always weakly higher than the wage of the family channel as long as there is some search for good jobs by the unemployed. Especially, if the homophily is complete ($\gamma = 1$), the two channels' wages are equal on average.

The intuition here is that if unemployed workers look for good jobs directly, the market arrival improves. This also implies that there will be more agents employed in good matches than mismatched. A consequence of this is that the performance of family increases with the homophily level: own type contacts are more likely to provide good offers since they are likely to be employed in good jobs. We show that only when the homophily is complete this latter effect is able to counterweight the direct search of the unemployed.

Next, we introduce professional contacts into the model as well. Here we identify two threshold values of the homophily level for any value of the unemployed search intensity and the number of professional contacts. These two thresholds correspond to the equality of expected wages between the market and the family channel on one hand, and between the market and the overall network (family+professional contacts) on the other hand. We show that actually the first threshold is higher than the second. This implies that for high enough homophily both the overall network and the family give better offers than the market, while for intermediate homophily levels only the overall network. Finally, for low values of homophily we always obtain wage discount for the overall network. We also derive that these threshold values are increasing in the unemployed search intensity level and decreasing in the number of professional contacts.

Here the intuition is that with the presence of professional contacts the number of good matches further increases making more likely that own type contacts will provide good offers. This means that the homophily level's effects are higher, and actually a high enough homophily level already implies that the family performs better than the market. Given that the overall network's expected wage also counts with the professional contacts, this causes that the overall network also provides wage premium and does so for lower homophily levels as well. On the other hand, when the homophily is very low, the unemployed search intensity makes the market expectation high enough to counterweight the effect of the work-related contacts.

The paper is organized as follows. The next session relates our work to the existing literature. In Section 2 we describe our model, derive the matching function and deduce the equilibrium conditions. Section 3 contains our analytical results while in Section 4 we perform numerical analysis and calibration to the US economy. Section 5 concludes.

1.2 Related literature

Our model is placed in the literature on social networks and labor market search and information flow. A closely related paper is Calvo-Armengol and Zenou (2005) which is built on the Pissarides framework and derives the matching function from assumptions on the information transmission process of the network. They focus on the effect of networks on unemployment rate whereas our model is focused on the mismatch in the society. Hence in our model workers and jobs are heterogenous and we include two types of relationships between agents: family and professional. [2] Other papers which concentrate on unemployment are Boorman (1975), Ioannides and Soetevent (2006), Calvo-Armengol and Jackson (2004). Calvo-Armengol and Jackson (2007) introduces heterogenous wages but they do not investigate mismatch since in their model workers are homogenous. In the paper of Boorman (1975) it appears the idea that some social contacts are privileged to get the information first (strong ties) while others have a secondary chance to do so (weak ties).

Bentolila et al. (2009) explicitly focuses on the effect of networks on mismatch, however

[2] Moreover we work in continuous time instead of discrete time which actually takes away the congestion effect producing their main result on non-monotonic relationship between connectivity and unemployment rate.

they assume that the market only transmits good offers while in our model it also provides bad ones. In fact, we have a parameter of unemployed search intensity toward good jobs, which varies the arrival rate of good and bad offers on the market. Further, they assume that contacts have information about different type of jobs at an exogenous rate. While in our model the state of the contacts is also endogenous, they might be unemployed or employed in different sectors which implies that they have access to different information. Hence we treat contacts in a symmetric way to the receiver of the information. We explicitly model the social networks as connections between agents, not only as an extra information channel for a given unemployed. This essentially changes the conclusions about mismatch since we do not obtain their negative result that networks always create mismatch.

The comparison of market and networks with respect to mismatch has mostly been investigated empirically and via comparing average wages of jobs found through formal search methods and social contacts. The most comprehensive paper is Pellizziari (2009) which uses the European Community Household Panel (ECHP) and finds mixed cross-country evidence: for Austria, Belgium and the Netherlands he estimates a wage premium of network search, while for Greece, Italy, Portugal and the United Kingdom he finds a wage discount. Partly using this data Bentolila et al. (2009) obtains wage discount for networks, however they restrict their attention to family contacts. On the other hand, a couple of papers finds that networks perform better than market in producing good matches (Simon and Warner (1992), Kugler (2003), Dustman (2010)). Our model is able to recover both wage premium and discount depending on the level of homophily within the family and the number of professional contacts.

Many papers investigate the effect of job referrals on the labor market outcomes (Montgomery (1991), Simon and Warner (1992), Tassier and Menzer (2001), Dustman (2010)), they underline the efficiency of social networks in producing good matches which comes from the role of referrals providing additional information compared to the market. Obviously this is a positive effect of networks which is represented by professional contacts in our model. However we also take into account favoritism (Bramoullé and Goyal (2009)) and show that, for some parameter range, the positive effect of networks on wages still holds in this case as well.

A somewhat related literature in sociology compares the quality of job information obtained through strong and weak ties. Granovetter (1995, originally 1974) obtains that work-related contacts provide better job offers than family contacts. This hypothesis has been extended under the name of "the strength of weak ties" which generated a huge literature. Marsden and Gorman (2001) summarizes this line of articles as it has produced mixed evidence on this relationship but still mostly it is supporting the original idea. In our paper we do not want to apply this labelling of the type of contacts, instead we prefer to distinguish professional and family contacts. The reason is that, even though the exact definition of weak and strong ties changes from one paper to another, the original idea is that strong ties have mostly access to the same information as the individual itself while weak ties have significantly different information. Here we rather want to emphasize that family contacts practice favoritism while professional contacts only recommend good candidates driven by caring about their reputation.

At last, there is a growing literature on the role of homophily, the tendency of similar individuals to be connected, in different processes on social networks (Golub and Jackson (2009), Currarini, Jackson and Pin (2009)). Homophily is usually seen as negative phenomenon since it is connected to segregation of different social groups (Moody (2001)). In particular, Golub and Jackson (2009) shows that some type of learning processes might be slowed down in a homophilous network because group boundaries reinforce the heterogeneity of behavior. On the other hand, being connected to similar agents might also be efficient since it also means more effective communication between individuals and easier access to relevant information. In fact, there is some evidence that links between similar agents tend to be more resistent to dissolution (McPherson et al. (2001)) indicating that these links are advantageous. In our model homophily is also positive in the sense that it increases the likelihood that an individual have access to information about offers of the sector where she is more productive. The dimension of similarity is occupation. We show that the degree of homophily within the family with respect to similarity in occupation is crucial in determining whether social networks or the market provide better paying jobs on average. To our knowledge, we are the first who explicitly focus on the effects of homophily on this question and on the labor market outcomes in general.

2 Model

Workers and Occupations. In this model we have two occupations (sectors) $s \in \{A, B\}$ and two types of workers $j \in \{A, B\}$. For simplicity we talk about the symmetric case when the mass of workers is equal between the two types: $\pi_A = \pi_B = 0.5$. Every worker is able to fill a job of any occupation but i workers have higher productivity in occupation i (good match) and lower productivity in occupation j (bad match), $i \neq j, i, j \in \{A, B\}$. The productivity in a bad match is p, in a good match $\bar{p}$, where $p < \bar{p}$. Unemployed workers earn b unemployment benefits ($b < p < \bar{p}$).

Network structure and information transmission. Every worker has k social contacts which consist of k_F family members (and friends) and k_P professional contacts ($k = k_P + k_F$). For simplicity we analyse the case where $k_F = 1$, ie. family consists of couples, and $k_P \geq 0$. The contact types refer to the nature of the relationship (tie), ie. an individual's professional contacts

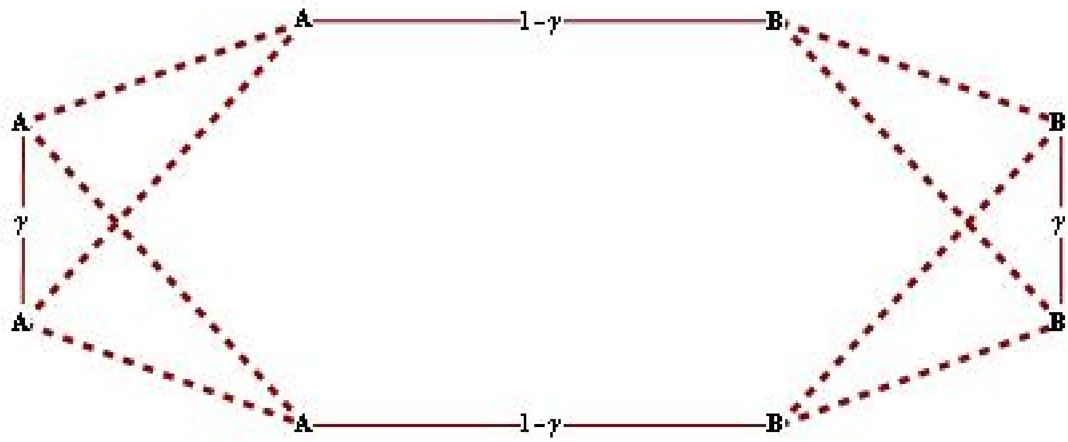

Figure 1: Network, $k_F = 1$, $k_P = 2$, solid line: family tie, dashed line: professional tie.

have their own family contacts as well.

The underlying network structure is a regular random graph with degree k. The probability that at the two ends of a family link two agents of the same type can be found is γ, consequently, that the two agents are of different types is $1 - \gamma$. γ measures the homophily in the society, ie. the tendency of individuals of similar characteristics to be connected. On the other hand, professional ties always connect similar agents. What we have in mind here is the idea that these ties are often formed in meetings, schools where similar type of agents meet or with the intention of having access to information of one's own profession. [3]

Figure 1 illustrates the described network structure. In this graph we have 4 type B and 4 type A agents. Every agent has 2 professional contacts (represented by the dashed lines) and 1 family contact (represented by the solid lines). Professional ties always connect similar type of agents, while family ties might connect different ones. The former happens with probability γ.

The difference between the contact types is the following: when an offer arrives to a worker who does not need that offer (because she is employed), first she checks with her family member whether he is unemployed and needs the offer (favoritism) and only in case there is no family need she transmits the information to professional contacts. In this second step, she only passes the offer if the professional contacts are of the appropriate type, hence a good match would be formed. The receiver of the information is randomly chosen among the unemployed professional contacts. If

[3] See McPherson et al. (2001) for evidence that individuals of the same occupation tend to be connected and that school and workplace are important places where these connections are formed. There is also wide evidence that family ties are homophilous along many dimensions.

there is no such, the offer is lost. Hence, the information might travel only one step in the network (as it was assumed in Calvo-Armengol and Zenou (2005) and in Calvo-Armengol and Jackson (2004)) which largely facilitates the analysis.

This formulation captures our original idea that family represents favoritism: family members consider each other as first receivers of information, however, family transmits information about bad matches as well (Bentolila et al. (2009)). On the other hand, professional contacts are considered only in the second round, but they pass only good information which exactly fits the characteristics of the workers. The reputation loss by recommending a low productivity worker is overruled by the value of having a family member employed which does not happen in the case of a professional contact.

For the derivation of the matching function we use the so-called homogenous mixing assumption which says that the probability that an individual is in some state (unemployed, mismatched or employed in the right sector) is equal to the population frequency of that state. This assumption basically means that the ties of the network are randomly drawn at each instant of time (Calvo-Armengol and Zenou (2005)). Hence, for example, the probability that a neighbor is unemployed is equal to u (the unemployment rate), and is independent of the state of any of her neighbors.

Observability of types. We assume that when a firm and an unemployed meet they both observe whether their match would constitute a bad or a good one. Thus we suppose that they accept bad matches because on a labor market with search frictions they would have to wait too much for a consecutive match. They both incure costs of waiting: for the firm to maintain the vacancy costs c, while a worker is better off to earn the wage corresponding to a bad match (w_B) than to have the unemployment benefit (b).

This latter is more likely if the productivity of a bad and a good match are close enough. In fact, in a two sector model of heterogenous workers Moscarini (2001) shows that workers whose productivity does not differ much between the two sectors search for and accept offers of both sectors. On the other hand, later we show that in this model firms earn the same by a bad and a good match, given that they pay less wages in the first case.

Given this assumption, our professional contacts are not exactly job referrals in the sense that they supply otherwise unknown information. Caring about their reputation they do not introduce bad matches for the employer, they serve as a good channel to contact appropriate workers.

Arrival of offers. Regarding the arrival of offers to individuals, we suppose a technology for each sector which describes the rate of direct arrivals. In a model without social networks this technology would be the matching function itself, however in our model we need to take into account that the offers are passed along the links of the network, hence the matching function is derived from the assumption on this transmission process.

We assume that the probability that an offer of sector $i \in \{A, B\}$ reaches someone in the society is given by:

$$m_i = Av_i^{\eta} \tag{1}$$

where v_i is the vacancy rate of the corresponding sector, A and η are technology parameters with the restriction of $0 < \eta < 1$. Hence, we assume decreasing return for the matches with respect to the vacancy rate: as there are more vacancies posted, the probability that a given vacancy is known by someone decreases ($m_i/v_i = Av_i^{\eta-1}$). Note that this technology does not depend on the unemployment rate (u), since not only unemployed hear about offers in this model.

Further, we assume that unemployed agents when they search have access to the offers of both sectors. However they might search more intensively for the offers of the sector where their productivity is higher. The probability that an offer of type i reaches an unemployed of type i is $u_i\theta Av_i^{\eta}$ where u_i is the probability that an individual of type i is unemployed and θ measures the search intensity for good jobs. If $\theta = 1$, the direct arrival is just random, if $\theta > 1$ unemployed direct their search. On the other hand, the probability that this unemployed is reached by a bad offer is $u_iAv_j^{\eta}$.

On the other hand, we suppose that when employed, individuals exclusively hear about the offers of the sector corresponding to their current occupation. This reflects the idea that they hear about new openings of their own firm or related firms in the sector. Since by the homogenous mixing assumption, the probability that an individual is employed in sector i is equal to the employment rate of this sector (e^i), the probability that an offer of type i reaches such an employed is $e^iAv_i^{\eta}$.

Note that in this way we exclude the possibility that a mismatched individual improves her position by direct arrival. On the other hand, by the formulation of the transmission of information by the contacts, we have also excluded the possibility that they hear about better offers indirectly. This is because we have assumed that workers having an unneeded offer consider only their unemployed friends as receiver of the information. Hence, in this model we disregard the possibility of on-the-job search.

Timing. We work in continuous time which implies that at some instant of time only one event can happen. Thus it cannot happen that an agent is employed and dismissed at the same instant of time, or that she receives a bad and a good offer at the same time, or that more than one information source provides offer for her at the same time.

2.1 Matching function

The technology defined in the previous section describes the number of "direct encounters" between vacancies and individuals. In our model, however, there is also a possibility to get information through social contacts: unemployed workers might hear about job openings through their employed friends. To construct the probabilities of job finding and vacancy filling we need to consider these two channels, direct and indirect. On the other hand, we define different probabilities for a bad and a good match to be formed. A bad match may occur through direct arrival (market) or family contacts, while for the good match we have an additional channel, the professional contacts.

2.1.1 Bad matches

A bad match occurs when an unemployed worker of type i receives an offer of type j ($i \neq j$). Among employed contacts only the ones employed in sector j may hear about offers of this sector. Given the assumption that every individual has only one family member, the probability that she passes information about a j offer is the following:

$$P_{bad}^{indirect} = Av_j^{\eta}(\gamma e_i^j + (1-\gamma)e_j^j) \tag{2}$$

where Av_j^{η} is the probability that she has an offer, γ is the probability that she is of type i, e_i^j is the probability that she is employed in sector j, $(1-\gamma)e_j^j$ is the probability that she is of the other type j and employed in sector j.

The probability that a given unemployed agent finds employment in the other sector takes into account the direct and indirect arrival as well. In continuous time these two events cannot happen at the same time:

$$q_B = Av_j^{\eta} + P_{bad}^{indirect} = Av^{\eta}(1 + \gamma e_i^j + (1-\gamma)e_j^j) \tag{3}$$

Hence, the probability that a new bad match is formed is just $u_i q_B$. This latter we may interpret as if Av_j^{η} was the probability that a vacancy reaches one of the individuals and $u_i(1+\gamma e_i^j + (1-\gamma)e_j^j)$ was the probability that this agent is an unemployed of type i or some employed in sector j with a family link to an unemployed of type i.

We may also construct the probability that a vacancy of sector j is filled by a bad match (an unemployed of type i):

$$q_B^F = u_i \frac{m_j}{v_j}(1 + \gamma e_i^j + (1-\gamma)e_j^j) = Av_j^{\eta-1}u_i(1 + \gamma e_i^j + (1-\gamma)e_j^j) \tag{4}$$

$\frac{Nm_j}{Nv_j}$ being the probability that a particular vacancy is matched to someone. Note that this is just $\frac{u_i q_B}{v_j}$.

2.1.2 Good matches

Here we consider the case that an unemployed worker of type i gets an offer of type i. The information source might be direct arrival, family and professional contacts.

First, we look at the event that the family contact transmits good job information, she might be of type i or type j:

$$P_G^{Family} = Av_i^\eta(\gamma e_i^i + (1-\gamma)e_j^i) \tag{5}$$

Next, we turn to professional contacts. They all are of type i. They pass information when they are employed in sector i and their family doesn't need it (they are employed). After they choose our given agent among their unemployed professional contacts at random. Putting together all this gives the probability that an offer reaches a given unemployed through a professional contact:

$$P_G^{Professional} = e_i^i Av_i^\eta(\gamma(1-u_i) + (1-\gamma)(1-u_j))\frac{1-(1-u_i)^{k_P}}{u_i k_P} \tag{6}$$

where e_i^i says that the contact is employed in the right sector, Av_i^η gives the probability that she has an offer. $(\gamma(1-u_i) + (1-\gamma)(1-u_j))$ is the probability that her family is employed being any type. The last term says the probability that our agent is chosen as receiver of the information among the k_P professional contacts of the sender who randomly picks one unemployed contact. This formula is derived from the binomial distribution with parameters (k_P, u):

$$\sum_{s=0}^{k_P-1}\binom{k_P-1}{s}u_i^s(1-u_i)^{k_P-1-s}\frac{1}{s+1} = \frac{1-(1-u_i)^{k_P}}{u_i k_P} \tag{7}$$

The sender of the information randomly draws k_P neighbors, each of them is unemployed with u_i, our given unemployed is chosen at random with probability $1/(s+1)$, where s is the number of competitors for the same information. Note that this formula is decreasing in the unemployment rate u since if there are more unemployed a given agent faces higher competition for the same information (see Boorman (1975), Calvó-Armengol and Zenou (2005)).

Since every agent has k_P professional contacts and at the same time only one of them might have information, the probability that some of them provides an offer is $k_P P_G^{Professional}$.

Now we aggregate the different sources of information: directly, family or professional contacts. In continuous time only one source can give information at the same time, hence we need to aggregate independent and exclusive events. The probability that a given unemployed receives an offer is given by:

$$q_G(u_i, u_j, v_i, e_i^i, e_j^i, e_i^j) = \theta A v_i^\eta + P_G^{Family} + P_G^{Professional} =$$
$$A v_i^\eta \left(\theta + \gamma e_i^i + (1-\gamma) e_j^i + k_P e_i^i (\gamma(1-u_i) + (1-\gamma)(1-u_j)) \frac{1-(1-u_i)^{k_P}}{u_i k_P} \right) = \quad (8)$$

where the first term in the sum stands for the direct arrival including the search intensity parameter θ. Then the probability that a match occurs is $u_i q_G$ and the probability that a vacancy of sector i is filled by a good match is just $q_G^F = \frac{q_G u_i}{v_i}$.

2.2 Value functions

In this section we write up the value functions associated to the states of the workers and the firms. Workers might be unemployed (U) or employed in a bad match (W_B) or in a good match (W_G). Firms post vacancies which might be vacant (V) or filled by a bad (J_B) or a good match (J_G). Since we focus on the symmetric equilibrium, we specify only value functions for bad and good matches and not for each possible encounters of types.

2.2.1 The worker

The discounted value of unemployment consist of the current value of unemployment benefit and the future value of possible employment which might be in the good or in the bad sector:

$$\delta U = b + q_B(W_B - U) + q_G(W_G - U) \quad (9)$$

where δ is the discount rate (common to workers and firms) and q_B and q_G represent the job finding rates of an unemployed, for a bad and a good match, respectively.

The discounted value of being employed in a bad match is the sum of the value of wages earned and the possibility of job destruction which happens at a rate λ in both bad and good matches.

$$\delta W_B = w_B + \lambda(U - W_B) \quad (10)$$

where w_B is the wage in a bad match.

On the other hand, the value function of being in a good match consist of the wages earned (w_G) and the future value of being unemployed:

$$\delta W_G = w_G + \lambda(U - W_G) \tag{11}$$

Note that it is optimal for the worker to accept a bad offer now if the discounted value of bad employment is higher than the value of staying unemployed and possibly get a good offer later. This is:

$$\delta(b + q_G(W_G - U)) \leq w_B + \lambda(U - W_B) \Longleftrightarrow q_G(W_G - U) - \lambda(U - W_B) \leq w_B - b$$

which says that the difference between the bad wage and the unemployment benefit should be high enough. We cannot solve this inequality to give conditions on the parameter values which would imply the optimality of the unemployed' decision. However, in the numerical part we ex-post compute this relation and verify that the unemployed indeed take the optimal decision (see the Table 5 in the Appendix).

2.2.2 The firm

Maintaining a vacancy is costly but it has the future benefits of becoming a job of either a bad or a good match. The value function is given by:

$$\delta V = -c + q_B^F(J_B - V) + q_G^F(J_G - V) \tag{12}$$

where c is the cost of vacancy and q_B^F and q_G^F are the job filling rates for bad and good matches, respectively.

The value of a bad match comes from the productivity of the worker in a mismatched job p and the future value of job destruction while wage has to be paid for the worker:

$$\delta J_B = p - w_B + \lambda(V - J_B) \tag{13}$$

A good match has the same value except that now the productivity of the worker is higher: $\bar{p} > p$ and that a different wage has to be paid:

$$\delta J_G = \bar{p} - w_G + \lambda(V - J_G) \tag{14}$$

Obviously, the wages paid in a bad match are lower compared to the wage of a good match which compensates the firm for receiving a lower productivity.

Due to the free entry condition for the firms, the competition ensures that the value of the vacancy reduces to zero: $V = 0$. Applying the zero-profit condition, we can express $J_B = \frac{p-w_B}{\delta+\lambda}$ and $J_G = \frac{\bar{p}-w_G}{\delta+\lambda}$. Substituting these into the equation (12), we get the job creation curve:

$$c = q_B^F \frac{p - w_B}{\delta + \lambda} + q_G^F \frac{\bar{p} - w_G}{\delta + \lambda} \tag{15}$$

2.3 Wage determination

Wages are determined by the usual Nash bargaining procedure upon meeting by workers and firms. We assume that the type of the worker and the type of the job is common knowledge between the firm and the worker, hence they condition their decision on the quality of the match.

The wage of a bad match is determined by the following:

$$w_B = \text{argmax}(W_B - U)^{\beta}(J_B - V)^{1-\beta}$$

where β is the bargaining power of the worker. This gives the usual first order condition:

$$W_B - U = \beta(J_B + W_B - V - U)$$

In the same way we obtain the FOC for the wage of the good match:

$$W_G - U = \beta(J_G + W_G - V - U)$$

Using the free-entry condition $V = 0$ and the expressions for J_B and J_G we may derive the wages from the FOC's, see the details in the Appendix.

The expressions for the wages are:

$$w_B = \frac{(1-\beta)b(\delta+\lambda) + (\delta+\lambda+q_B)(p-(1-\beta)\bar{p}) + \beta p q_G}{\delta+\lambda+\beta(q_B+q_G)} \tag{16}$$

$$w_G = \frac{(1-\beta)b(\delta+\lambda) + (p-\bar{p})q_B + \beta(-pq_B + \bar{p}(\delta+\lambda+2q_B+q_G))}{\delta+\lambda+\beta(q_B+q_G)} \tag{17}$$

Note that the difference between the two wages is just the productivity difference $\bar{p} - p$. This implies that the value of a filled vacancy in a bad and a good match is equal in equilibrium: $J_B = J_G$. This is because in equilibrium every firm should earn the same zero profits due to the free-entry condition. Hence when facing a bad match they pay a reduced wage and completely shift their losses to the workers. This also implies that they are indifferent in accepting a good or a bad match.

Both wages are increasing in q_B and q_G, higher job finding probability means better outside options for the worker.

2.4 Equilibrium

We write down the conditions defining the stationary equilibrium of the model.

Throughout in our analysis we assume that the two types of workers have the same share in the population and are equally homophilous. They have the same number of connections, job separation rates and the productivity difference between good and bad matches are also the same for the two groups. In this case their situation is symmetric: they will have the same unemployment rate, employment rates in the good and bad types of jobs. Hence, the firms will have the same incentives to enter in the two sectors. This implies the following equalities:

- $v_i = v_j \equiv v$
- $u_i = u_j \equiv u$
- $e_i^j = e_j^i \equiv e_B$
- $e_i^i = e_j^j \equiv e_G$

Note that, we have some accounting identities: $1 - u_i = e_i^j + e_i^i$ and $1 - u_j = e_j^j + e_j^i$, so in the symmetric case: $1 - u = e_B + e_G$.

Applying these equalities, we write down the set of equations defining the equilibrium. First the job finding probabilities:

$$q_B(u, v, e_B) = Av^\eta(1 + \gamma e_B + (1 - \gamma)(1 - u - e_B)) \tag{18}$$

Note that this probability is decreasing in u and increasing in e_B if $\gamma > 0.5$ while decreasing if $\gamma < 0.5$. On the other hand, the effect of the homophily parameter depends on the state of the economy: if the mismatch is high, homophily increases the probability of receiving a bad offer: same type agents are most probably employed in the bad sector. On the other hand, if the mismatch is low, own type contacts are in good employment, hence they pass information about good matches.

$$q_G(u, v, e_B) = Av^\eta\left(\theta + \gamma(1 - u - e_B) + (1 - \gamma)e_B + (1 - u - e_B)(1 - u)\frac{1 - (1 - u)^{k_P}}{u}\right) \tag{19}$$

The probability of receiving good job information is decreasing in u since unemployed are competitors for the same information. It also decreases in e_B for $\gamma > 0.5$: if there are many agents in bad jobs they will learn mostly about offers of the bad sector hence the probability of receiving a good offer is lower. The effect of γ again depends on the state of the economy. Homophily increases the probability to hear about a good offer if there are more people employed in the good sector than in the bad. This probability increases with the number of professional contacts k_P since they provide good information only.

Regarding steady state turnover: in steady state the in- and outflows to employment have to be equal both for good and bad matches. At any moment of time a negative shock may arrive according to a Poisson process with parameter λ. Upon arrival the productivity of the match decreases such that the firm exits the market and the job is finished. We have the following two turnover equations for bad and good matches, respectively:

$$\lambda e_B = u q_B(u, v, e_B) \tag{20}$$

$$\lambda(1 - u - e_B) = u q_G(u, v, e_B) \tag{21}$$

The Job Creation equation has been derived in the previous section, this uses the solutions of the wage bargaining problem.

$$c = \frac{u q_B(u, v, e_B)}{v} \frac{p - w_B}{\delta + \lambda} + \frac{u q_G(u, v, e_B)}{v} \frac{\bar{p} - w_G}{\delta + \lambda} \tag{22}$$

Wages:

$$w_B = \frac{(1-\beta)b(\delta + \lambda) + (\delta + \lambda + q_B)(p - (1-\beta)\bar{p}) + \beta p q_G}{\delta + \lambda + \beta(q_B + q_G)} \tag{23}$$

$$w_G = \frac{(1-\beta)b(\delta + \lambda) + (p - \bar{p})q_B + \beta(-p q_B + \bar{p}(\delta + \lambda + 2q_B + q_G))}{\delta + \lambda + \beta(q_B + q_G)} \tag{24}$$

The equilibrium is defined as follows.

Definition 1 *The equilibrium of the model is a triple* $\{u^*, e_B^*, v^*\}$ *satisfying the equations (20), (21), (22).*

The following proposition states that under some conditions the equilibrium is unique.

Proposition 1 *Assume that*

1. $(-1 + \eta)\lambda + A(-1 + 2\gamma) < 0$
2. $\gamma\lambda \geq A(1 - (1-\gamma)(2\gamma - 1))$
3. $\bar{p} - p \leq \frac{\beta(\bar{p} - b)}{\beta + \delta + \lambda}$.

Then if equilibrium exists, it is unique.

Proof See Appendix ∎

Note that here we face the problem that the arrival rate of offers positively depends on the employment level which actually increases in the arrival rates itself. This is because we have assumed that the arrival of offers to employed agents depends on their current occupation and that the employment rate in the society is equal to the probability that a given agent is employed. This would possibly create multiple equilibria in the model. However, we have managed to give conditions for which this does not happen and this makes the following comparative statics possible.

Under the conditions detailed in the proposition the Job Creation curve of the model is upward sloping whereas the Beveridge curve is downward sloping in the (u, v) plane. We were not able to prove that they actually cross, but the numerical analysis shows that this is the case and our analytical results show the intuition for the comparative static results.

3 Results

Our main question is whether social networks perform better or worse in matching the workers and jobs of similar characteristics compared to the market process. To this end we compare the expected wages associated to the offers coming from the direct arrival and the network channel. We define the expected wages of formal search as follows.

Definition 2 *The expected wage of the formal search is the conditional expectation of the wage which is obtained through direct arrival condition on the event that an offer has arrived:*

$$\frac{Av^{\eta}w_B + A\theta v^{\eta}w_G}{(1+\theta)Av^{\eta}} = \frac{w_B + \theta w_G}{1+\theta}$$

Hence if the unemployed does not search for good jobs ($\theta = 1$), in symmetric equilibrium the direct arrival provides good and bad offers with the same probability and the expected wage is just the average of the wages in good and bad matches. On the other hand, if $\theta > 1$, the expectation puts higher weight on good jobs.

As for the network channel we define two expectations, one for the family contacts only and an other for the family and professional contacts together. The expected wage of the network search using family members for an unemployed worker:

Definition 3 *The expected wage coming through the family is defined as the conditional expectation of the wage obtained through the family member conditional on the event that an offer has* *arrived through this contact:*

$$\frac{q_B w_B + (\gamma e_G + (1-\gamma)e_B)w_G}{q_B + \gamma e_G + (1-\gamma)e_B} = \frac{(\gamma e_B + (1-\gamma)e_G)w_B + (\gamma e_G + (1-\gamma)e_B)w_G}{e_B + e_G}$$

On the other hand, considering both family and professional contacts, we have the following expectation.

Definition 4 *The expected wage of network search is the conditional expectation of the wage obtained through the network channel (family+professional contacts) contidional on the event that some offer has arrived through this channel:*

$$\frac{Pr_B^N w_B + Pr_G^N w_G}{Pr_B^N + Pr_G^N}$$

where $Pr_B^N = \gamma e_B + (1-\gamma)e_G$ *and* $Pr_G^N = (\gamma e_G + (1-\gamma)e_B) + e_G(1-u)\frac{1-(1-u)^{k_P}}{u}$.

The expected wage of the network search is the conditional expectation of the wage coming through the network channel (family or professional) conditioned on the event that some offer came through it.

We are interested how the three important parameters of the model (θ - search for good jobs by the unemployed, k_P - number of professional contacts, γ - homophily) affect these quantities. Apart from this issue we are also concerned how other endogenous variables of the model (unemployment and vacancy rates, wages) change by varying these parameters.

First, we concentrate on the case where there are no professional contacts ($k_P = 0$) and we compare the market with the network of family members. Later we introduce professional contacts, it turns out that there is no qualitative difference between the case when the number of the two types of contacts is equal or there are more professional contacts.

In this section, we derive some analytical results while in the next one we calibrate the model to the US economy.

3.1 Model without professional contacts

In the first case, we investigate only the effects of family on the mismatch in the economy. We compare the expected quality of offers coming through the market and family and shall see the effects of the homophily (γ) and unemployed search (θ) on this.

The first proposition compares family and the market when the unemployed do not search for good jobs ($\theta = 1$).

Proposition 2 *Without professional contacts ($k_P = 0$) and unemployed directed search ($\theta = 1$),*

(i) in equilibrium the fraction of workers employed in bad and good matches are equal ($e_B = e_G$),

(ii) both the market and the family provide bad and good matches uniformly random,

(iii) the homophily has no impact on the mismatch and the relative performance of market and family networks.

Proof See Appendix. ∎

The intuition behind this result is based on the assumption that employed individuals learn about the offers which correspond to their sector of occupation. Obviously, if $\theta = 1$, the direct arrival provides good and bad jobs with equal probability. As for the family network, if the employment rates in good and bad matches are the same, an employed of any type has equal probability to be employed in any kind of job. This implies that the arrival rate of information about bad and good matches through contacts and overall between the two sources of information will be the same. A direct consequence of this equality is that the employment rates will be equal as well and that homophily has no effect on the equilibrium since own type contacts might be mismatched or in good employment with equal probability.

Note that as long as there is no on-the-job search in the society we obtain the same result if the arrival of offers to employed agents is random as well (just as for unemployed agents). In that case employed contacts send any offers they hear about which is equally likely to be bad or good.

In the Appendix we show that this Proposition still holds with many family contacts as long as having an offer they randomly choose which unemployed family member receives the information (that is, they do not direct offers to good matches which would create a positive bias for family).

If we introduce unemployed search for good jobs ($\theta > 1$), we create a positive bias in the direct arrival process which implies that there will be more agents in good employment than in bad ($e_G > e_B$). In this case homophily starts to affect the equilibrium and the expected wage of the family channel: it increases the ratio of good employed (e_G/e_B) and the family wage expectation (see equations (18) and (19)). To be connected to an own type employed increases the probability of receiving a good offer since this contact has higher probability to be employed in a good job than to be mismatched. However, the following proposition says that the effect of homophily is never strong enough to make the expected wage of the family channel to be higher than the expected wage of the direct arrival (market). In fact, at complete homophily ($\gamma = 1$), the two channel's expected wages coincide for any level of unemployed search θ.

Proposition 3 *Without professional contacts ($k_P = 0$), if the unemployed search for good jobs ($\theta > 1$),*

(i) there are more employed in good jobs than in bad ($e_G > e_B$), the difference increases with θ and the homophily (γ),

(ii) the market provides weakly better offers on average than the family network,

(iii) homophily (γ) increases the expected wage of family, especially, if $\gamma = 1$, market and family provides good offers at the same rate for any value of θ.

Proof See Appendix. ∎

Observe that the derivative of the weight of high wages in the expectation of the wage of family search with respect to γ is $e_G - e_B$ (see definition 3). Hence homophily affects more the equilibrium if the difference in good and bad employment increases. This would imply that as θ gets higher, there might be some chance that increasing homophily the family wage expectation becomes higher than the market expectation. However, as it is shown, this effect is always overuled by the increase of market expectation as θ gets larger. Hence, there is no θ value such that increasing the homophily to high enough, we obtain strictly higher expected wage for the family than for the market. However, interestingly, if the homophily is complete $\gamma = 1$, for any value of unemployed search, the two search methods perform equally well. [4]

Note that Bentolila et al. (2009) has obtained the same result that the network perform strictly worse except for the case of complete homophily. However, they assumed that on the market unemployed exclusively look for good jobs, while in our model this holds for any value of the unemployed search intensity θ.

3.2 Model with professional contacts

Introducing professional contacts obviously attributes positive properties to the social networks: it adds a new channel to receive offers and it is guaranteed that this channel provides information about good matches. The relative efficiency of markets and networks (professional+family contacts) depends on the relative magnitude of the unemployed search (θ) and the number of professional contacts.

[4] Note that perfect homophily ($\gamma = 1$) does not mean that contacts only provide good offers. Since with probability $e_B > 0$ they are employed in bad sector, they have chance to send information about bad jobs.

First we show that with positive number of professional contacts or with unemployed directed search, the fraction of employed in good jobs is higher than the ratio of mismatched. This also implies that homophily will increase the number of good matches and the performance of the social networks.

Proposition 4 *For any positive number of professional contacts ($k_P > 0$) or unemployed directed search ($\theta > 1$), in equilibrium the fraction of workers employed in good jobs is higher than the fraction of employed in bad jobs ($e_G > e_B$). The difference is increasing in k_P, in θ and in the homophily index γ.*

Proof See Appendix. ∎

Once unemployed direct their search ($\theta > 1$) or there are professional contacts ($k_P > 0$), the overall arrival process of offers is biased toward good offers, hence in equilibrium there will more individuals employed in good jobs than mismatched. As soon as $e_G > e_B$, the homophily further improves the arrival of good offers: it is better to be connected to own type contacts since they have higher chance to be in good employment and have a good offer. This improvement obviously comes through the network channel.

This proposition also implies that if the market arrival is random ($\theta = 1$), the network search performs better than the market for $k_P > 0$ since in this case the only positive bias is attributed to the networks. In particular if $\gamma > 0.5$, the family itself also provides better information than the market.[5]

It is more interesting to observe what happens if the unemployed direct their search toward good offers ($\theta > 1$), which means that the market arrival is biased as well. The following Proposition shows that essentially the homophily index is decisive whether the family network or the overall network performs better than the market in terms of providing good matches.

Proposition 5 *If professional contacts are present ($k_P \geq 1$), for any value of the unemployed search θ,*

(i) there exist a homophily value $\bar{\gamma}_1$ such that for any $\gamma \in (\bar{\gamma}_1, 1]$ the family contact provides higher expected wage than the market,

(ii) $\bar{\gamma}_1$ is decreasing in the number of professional contacts,

[5]This can be seen on the weight of high wages in definition (3): if $\gamma = 0.5$, it is 0.5, as in the case of market. However, if $e_G > e_B$, it is increasing in γ, hence it goes above 0.5 if $\gamma > 0.5$.

(iii) There exists an interval $(\bar{\gamma}_2; \bar{\gamma}_1]$ such that for any γ in this interval the market expected wage is higher than the family one but lower than the overall social network's expected wage,

(iv) $\bar{\gamma}_2$ is decreasing in the number of professional contacts if $\theta \geq 2$.

Proof See Appendix. ∎

In the model without professional contacts the indifference homophily level between the market and family expected wage was the complete homophily ($\gamma = 1$) for any value of the unemployed search intensity θ. However, when there are some professional contacts (even only one), the indifference homophily level is strictly below 1. This is because the effect of the homophily parameter depends on the difference $e_G - e_B$ which is high enough if we introduce even only one professional contact. This is true for any level of θ. If we increase the search intensity θ, not only the market's performance increase but the $e_G - e_B$ difference as well which makes the effect of homophily higher. In the case without professional contacts the first effect dominated the second one, but introducing some bias in the network arrival makes the second effect dominant.

From here it is understandable that as the professional contacts become more numerous, the threshold homophily level decreases.

On the other hand, for any homophily level, the overall networks (family+professional contacts) perform strictly better than the family since the professional contacts provide only good offers. The expected wage of the overall networks increases in the same way with the homophily level as the family expectation. This gives rise to an interval of homophily level where the family performs worse than the market while the overall networks still performs better. If the homophily is low enough, the market outperforms the social networks when unemployed agent direct their search a little bit.[6]

In the following section the numerical analysis reveals that the threshold homophily levels are increasing in the search intensity parameter θ. Hence, the original intuition that in the end the relative performance of the market and social networks depends somehow on the ratio of θ and k_P is true. However, as our analysis points out, these parameters operate through the homophily level, by moving the threshold for family and/or network wage premium over the market. For any value of θ and $k_P > 0$, we always can find a high enough homophily level such that not only the overall network but also the family part of it provides better offers than the market.

This observation indicates the importance of homophily in the labor markets. To further underline this finding, we show that for low enough homophily level (where γ does not guarantee

[6] In the worst case $\gamma = 0$, here the weight on good wages in the network expectations is: $\frac{x+P}{1+x+P}$ which is lower than the market's weight if $x + P < \theta$, a sufficient condition is $\theta > 2$.

a wage premium for the overall network), for any number of professional contacts, we always can find a search intensity level such that the market's wage expectation is higher than the networks' one.

Proposition 6 *If the homophily level is low enough ($\gamma < \bar{\gamma}_2$), for any number of the professional contacts k_P there exists a value $\bar{\theta}$ such that for any $\theta > \bar{\theta}$ the market provides higher expected wage than the network (family + professional contacts).*

Proof See Appendix. ∎

Summing up, we may distinguish 3 cases for a given number of the search intensity θ and homophily level γ. First, the number of professional contacts is low such that the two threshold of homophily are close to 1. In this case unless the network exhibits perfect homophily, both the family contacts and the overall networks give a wage discount compared to the market. Second, adding more professional contacts moves down the homophily thresholds, hence the overall networks give a wage premium but the family still may give a wage discount. Third, if there are many professional contacts, both homophily thresholds are at a low level, hence both family and the overall network provides good offers at a higher rate than the market.

Finally, we turn to the effects of these parameters on other endogenous variables, The following proposition says that the number of professional contacts and the unemployed search intensity for good jobs make the unemployment rate to decrease. As the network gets denser, the job transmission process is more effective, and the arrival rate of offers gets higher. Note that in our case of continuous time we do not obtain the same result as Calvo-Armengol and Zenou (2005). In their model after a certain threshold the denser network means higher unemployment which is explained by the congestion effect, ie. many offers reach the same unemployed which are not passed further. In continous time this congestion effect is absent.

Proposition 7 *If the number of professional contacts k_P or the unemployed search intensity (θ) increases, the equilibrium unemployment rate u^* decreases.*

Proof See Appendix. ∎

Some other comparative static results are detailed in the next section.

4 Numerical analysis: calibration to the US economy

4.1 Calibration

In this section we calibrate the model for the US economy at a monthly frequency and try to assess how much is the effect of professional contacts and homophily on the level of mismatch in the economy and whether the network search yields wage discount or premium for the empirically relevant parameter values. We are also concerned how the threshold values of θ and γ as identified in the previous sections depend on other parameters of the model. Finally, we will also investigate what is the effect of these parameters on other endogenous variables such as the vacancy rate and wages.

Table 1 summarizes the parameter values and the reasons for the choice. The bargaining power of the workers β is set to 0.5, a value mostly used by the literature (Mortensen and Pissarides (1999), Fontaine (2008)). The discount rate δ is set to 0.988 which corresponds to a monthly interest rate of $r = 0.005$ used by Fontaine (2008). Following Shimer (2005) we set the value of the leisure b to 0.4 which ex-post gives an unemployment benefit replacement ratio around 53% in the benchmark case (fitting well the empirically relevant value which is 50-55%).

As for the monthly separation rate λ, we use the value of 3.4 % as estimated by Shimer (2005). The productivity in a good match $\bar{p}$ is normalized to 1. The productivity of the bad match p is computed to fulfill condition 3 of Proposition 1 which given the mentioned parameter values says that p should be higher than 0.8522. Taking this into account we set $p = 0.86$. The value of the cost of posting a vacancy is set to $c = 0.3$, a value used by Fontaine (2008).

The value of A, the technology parameter in the matching function is determined to fulfill condition 2 of Proposition 1 for our benchmark homophily level $\gamma = 0.8$. The condition says that $A \leq 0.031$, hence we set $A = 0.023$. Note that later we change the level of homophily since this is one of our parameters of interest. The other matching parameter η is taken to be 0.3 which is the lower bound of the estimates in Petrongolo and Pissarides (2001) (from 0.3 to 0.5). By this value condition 1 of Proposition 1 is satisfied for $\gamma = 1$, the most restrictive value for this condition.

Finally, we calibrate the value of θ to match the US unemployment rate averaged between 1951 and 2003, $u = 5.65\%$ (Shimer (2005)). This gives $\theta = 66.05$ at 10 professional contacts ($k_P = 10$) and $\gamma = 0.8$, our benchmark values of the parameters of interest. Solving the model with these values we obtain a vacancy rate of 2.4%, which gives a market tightness (v/u) of 0.424 which is close to the empirically observed value being around 0.5 averaging the last 10 years (see the JOLTS and CPS statistics of the BLS).

Parameter	Value	Reason
β	0.5	Standard in the literature
γ	0.8	benchmark, varied later
δ	0.988	monthly interest rate $r = 0.005$, Fontaine (2008)
b	0.4	Shimer (2005)
$\bar{p}$	1	Normalization
p	0.86	Condition 3 of Prop 1
c	0.3	Fontaine (2008)
λ	0.034	estimated by Shimer (2005)
k_P	10	benchmark, varied later
A	0.023	Condition 2 of Prop 1
η	0.3	Condition 2 of Prop 1, Petrongolo and Pissarides (2001)
θ	66,05	Calibrated to match $u = 5,65\%$

Table 1: Parameter values

4.2 Results

In this section we solve the model numerically using the calibrated parameter values. We show how the number of professional contacts and homophily affects the equilibrium by the calibrated value of the unemployed search intensity θ. Later, we demonstrate the threshold values of homophily for different k_P's and θ's.

The left panel of Figure 2 shows the expected wage difference between the family and the market arrival for different homophily values and number of professional contacts. We may see that family gives a wage discount compared to the market for most values of the parameters. The only slight wage premium is when the homophily is complete ($\gamma = 1$). On the other hand, the family expectation improves with the homophily parameter while it is not affected much by the number of professional contacts. This suggests that at the calibrated value of the unemployed search intensity (θ), the homophily threshold which makes the family performing better than the market is very close to 1 (as it is confirmed in Table 2, see later).

On the other hand, the right panel of Figure 2 plots the wage difference between the overall network (family+professional contacts) and the market. Here we see that obviously with the number of professional contacts the network expected wage gets larger. However, at the calibrated value of θ the network is able to give a wage premium only for the higher values of the homophily index. For $\gamma = 0.6, 0.7$ even for 30 professional contacts the overall network performs worse than

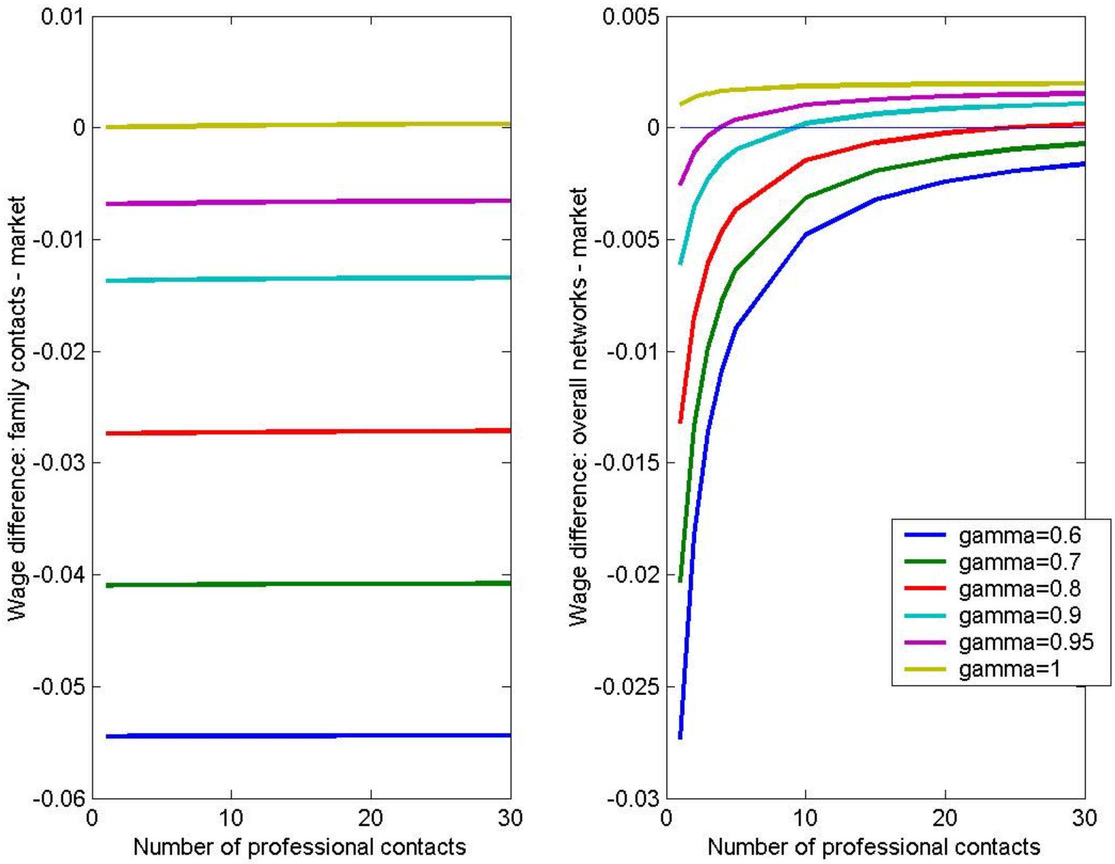

Figure 2: Wage difference between family and market (left panel) and between networks (family+professional) and market (right)

the market. [7] However, homophily increases the expected wage of the network for any number of professional contacts. Especially, if γ is closer to 1, even for small values of k_P the network gives a wage premium.

To illustrate Proposition 5, we have computed the thresholds of the homophily parameter γ for different values of the unemployed search intensity and professional contacts. Table 2 indicates the thresholds by which the family contacts and the market gives the same expected wages. We may see that the threshold is increasing in θ and decreasing in k_P. However, it is fairly close to but strictly less than 1 for all values.

On the other hand, Table 3 shows the threshold homophily index which equalizes the overall

[7]The same is the case if we increase k_P to 50

θ/k_P	1	2	3	4	5	10	15	20	25	30
2	0.9721	0.9615	0.9573	0.9557	0.9550	0.9546	0.9546	0.9546	0.9546	0.9546
5	0.9897	0.9837	0.9802	0.9780	0.9766	0.9747	0.9745	0.9745	0.9745	0.9745
10	0.9956	0.9926	0.9903	0.9887	0.9874	0.9845	0.9837	0.9836	0.9835	0.9835
30	0.9992	0.9985	0.9979	0.9974	0.9970	0.9954	0.9946	0.9940	0.9937	0.9935
66.05	0.9998	0.9996	0.9995	0.9993	0.9992	0.9986	0.9982	0.9979	0.9976	0.9975

Table 2: Threshold values of the homophily parameter: family wage=market wage

network and market expectations. First, as indicated by the proposition, these thresholds are lower than the ones from the previous table for each values of the other parameters θ and k_P. In the same way, it is increasing in the unemployed search intensity and decreasing in the number of professional contacts. If unemployed agents do not search much for good jobs but the professional contacts are used, the homophily threshold is quite moderate but still higher than 0.5. On the other hand, for the calibrated value of θ, the threshold is higher if there are only few professional contacts.

Table 4 illustrates Proposition 6: we have computed thresholds of the unemployed search intensity for some values of homophily and professional contacts. Note that for complete homophily $\gamma = 1$ such a threshold does not exists as implied by Proposition 5. Also, if $\gamma = 0.95$ and there is a considerate number of professional contacts, the threshold θ is very high. In general, $\bar{\theta}$ increases with both of the other parameters.[8]

θ/k_P	1	2	3	4	5	10	15	20	25	30
2	0.7399	0.6338	0.5906	0.5732	0.5662	0.5617	0.5616	0.5616	0.5616	0.5616
5	0.8778	0.8028	0.7561	0.7269	0.7087	0.6822	0.6800	0.6798	0.6798	0.6798
10	0.9265	0.8714	0.8296	0.7974	0.7727	0.7124	0.6966	0.6926	0.6916	0.6914
30	0.9706	0.9445	0.9212	0.9003	0.8815	0.8114	0.7682	0.7414	0.7249	0.7148
66.05	0.9858	0.9724	0.9598	0.9479	0.9366	0.8886	0.8513	0.8220	0.7990	0.7807

Table 3: Threshold values of the homophily parameter: overall network wage=market wage

We also investigate how the two network parameters, γ and k_P affect the other endogenous variables of the model, like unemployment and vacancy rates and wages (see Figure 3). We may see that the unemployment rate is getting lower as there are more professional contacts. Obviously,

[8]Note that this table shows the same relationship as Table 3. For instance, if $\theta = 2$ and $k_P = 1$, $\bar{\gamma} = 0.7399$ in Table 3. In the other table this means that for $k_P = 1$ and $\gamma < 0.7399$, the θ threshold is less than 2.

γ/k_P	1	2	3	4	5	10	15	20	25	30
0.6	1.510	1.846	2.052	2.168	2.228	2.279	2.280	2.280	2.280	2.280
0.7	1.789	2.520	3.208	3.872	4.524	7.817	11.230	14.560	17.639	20.391
0.8	2.602	4.878	7.503	10.252	13.041	26.961	40.557	53.795	66.695	79.279
0.9	6.616	14.280	22.074	29.869	37.642	76.092	113.882	151.102	187.823	224.100
0.95	16.194	33.903	51.608	69.264	86.868	174.168	260.473	345.978	430.808	515.051

Table 4: Threshold values of θ, overall network wage=market wage

the matching process is more effective if the network is more connected. On the other hand, the homophily parameter has no effect on the unemployment rate, it better determines the type of the job than the arrival rate of offers. Direct consequence of the previous analysis is that the fraction employed individuals in bad jobs is decreasing with homophily and the number of work-related contacts. The two last panels show that the wages increase with k_P but are unaffected by the homophily index. This is because as the unemployment rate gets lower, the outside option of the workers improves which makes their bargaining power higher. Hence they are able to claim higher wages.

As for the vacancy rate, if the unemployment reduces, the job filling rate of a given vacancy decreases as well, which put lower incentives for vacancy posting. The wage increase has the same effect. Hence, with the increase of the number of work-related contacts the firms post less vacancies.

5 Conclusions

We have investigated the question what is the effect of social networks on mismatch of workers to occupations, whether networks perform better or worse than the market in this respect. In our model individuals have two types of relationships: family contacts who favor each other and pass any offer even though it does not fit the abilities of their family member, and professional contacts who forward only good offers caring about their reputation. Family ties connect similar type of agents with a certain probability γ which is the homophily index.

We have shown that in the model without professional contacts if the unemployed agents direct their search to some extend toward good jobs, then the market always performs weakly better than the network. In particular, we only obtain equality between the expected wages of the two methods when the homophily is complete ($\gamma = 1$).

On the other hand, if we introduce professional contacts to the model, we obtain two thresh-

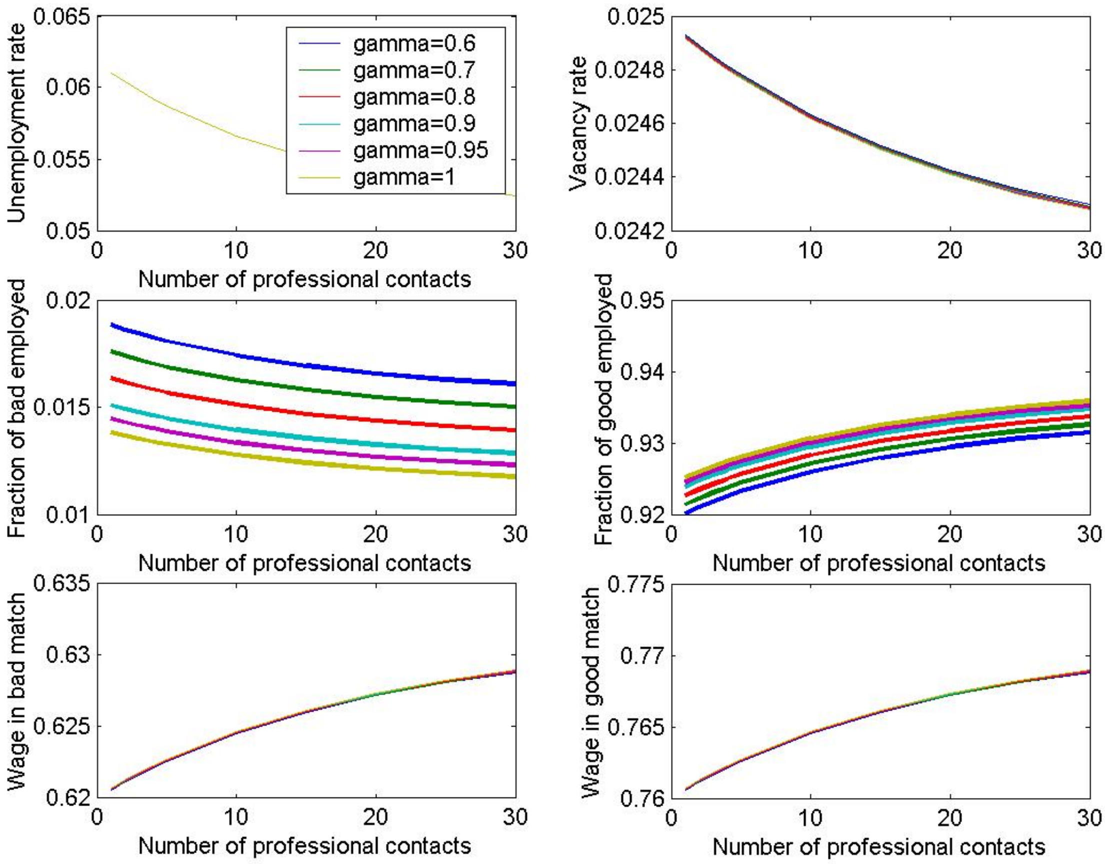

Figure 3:

old values of the homophily index which separates three different cases. First, for high a enough homophily level we always have that both the family network and the overall network (family + professional contacts) provide better offers than the market. Second, for intermediate homophily levels only the overall network performs better than the market, the family yields a wage discount. Third, for a sufficiently low homophily level, the market gives a higher expected wage than any of the two types of contacts.

These findings hold for any value of the unemployed search intensity and number of professional contacts which underlines the crucial role of homophily regarding the comparison between social networks and markets. However, we have seen that the threshold values of the homophily index change with other parameters of the model. In fact, as unemployed agents search more intensively for good jobs on the market the critical homophily levels rise. The same happens if the network of professional ties is sparsely connected.

6 References

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7 Appendix

7.1 General k_F

We may extend the result of section 3.1 to any number of family contacts. We assume that upon being aware of a new offer, a contact transmits the information to one of their unemployed family member chosen at random. This means that they do not direct their offer to family members of the type of the offer, they care only about the employment of their contacts, not about the type of the match. Under this assumption the probability of receiving a good or a bad offer coincides:

$$q_B = Av^\eta \left(1 + k_F(1-\gamma)e_G \frac{1-(1-u)_F^k}{uk_F} + k_F\gamma e_B \frac{1-(1-u)_F^k}{uk_F}\right)$$

and

$$q_G = Av^\eta \left(1 + k_F\gamma e_G \frac{1-(1-u)_F^k}{uk_F} + k_F(1-\gamma)e_B \frac{1-(1-u)_F^k}{uk_F}\right)$$

which are clearly the same when $e_B = e_G$.

7.2 Calculation of wages

Using (10) and (11), we can express the value function of the workers when being employed:

$$W_B = \frac{w_B + \lambda U}{\lambda + \delta} \tag{25}$$

$$W_G = \frac{w_G + \lambda U}{\lambda + \delta} \tag{26}$$

Now, we can express the value function U as function of the two wages:

$$U(w_B, w_G) = \frac{b(\lambda + \delta) + q_B w_B + q_G w_G}{\delta(\delta + \lambda + q_B + q_G)}$$

Substituting this into the equation W_B and W_G, these become functions of wages as well.

The two first-order conditions which define the wages are:

$$(1 - \beta)(W_B - U) = \beta J_B$$

$$(1 - \beta)(W_G - U) = \beta J_G$$

where everything is written as the function of the two wages (remember, $J_B = \frac{p - w_B}{\delta + \lambda}$ and $J_G = \frac{\bar{p} - w_G}{\delta + \lambda}$).

Solving the system we obtain the wages:

$$w_B = \frac{(1 - \beta)b(\delta + \lambda) + (\delta + \lambda + q_B)(p - (1 - \beta)\bar{p}) + \beta p q_G}{\delta + \lambda + \beta(q_B + q_G)} \tag{27}$$

$$w_G = \frac{(1 - \beta)b(\delta + \lambda) + (p - \bar{p})q_B + \beta(-p q_B + \bar{p}(\delta + \lambda + 2q_B + q_G))}{\delta + \lambda + \beta(q_B + q_G)} \tag{28}$$

Some derivatives:

$$\frac{\partial w_B}{\partial q_B} = \frac{\partial w_G}{\partial q_B} = \frac{(-1 + \beta)((\delta + \lambda)(\beta(b - \bar{p}) + \bar{p} - p) + \beta(\bar{p} - p)q_G)}{(\delta + \lambda + \beta(q_B + q_G))^2}$$

This is positive only if $(\delta + \lambda)(\beta(b - \bar{p}) + \bar{p} - p) + \beta(\bar{p} - p)q_G \leq 0$, a sufficient condition for this:

$$\bar{p} - p \leq \frac{\beta(\bar{p} - b)}{\beta + \delta + \lambda}$$

$$\frac{\partial w_B}{\partial q_G} = \frac{\partial w_G}{\partial q_G} = \frac{(-1 + \beta)\beta((b - \bar{p})(\delta + \lambda) + q_B(p - \bar{p}))}{(\delta + \lambda + \beta(q_B + q_G))^2}$$

which is clearly positive since $p < \bar{p}$ and $b < \bar{p}$.

Note that the wage difference $w_G - w_B$ is equal to the productivity difference $\bar{p} - p$.

We also have that the wages react more to the change of q_G than to change of q_B:

$$\frac{\partial w}{\partial q_G} - \frac{\partial w}{\partial q_B} = \frac{(1-\beta)(\bar{p}-p)}{\delta + \lambda + \beta(q_B + q_G)} > 0$$

This is because the value of the outside option increases more if the chance to be hired in a good match increases compared to be hired in a bad match.

7.3 Proofs

7.3.1 Proposition 1

Assume that

- $(-1+\eta)\lambda + A(-1+2\gamma) < 0$
- $\gamma\lambda \geq A(1-(1-\gamma)(2\gamma-1))$
- $\bar{p} - p \leq \frac{\beta(\bar{p}-b)}{\beta+\delta+\lambda}$.

Then if equilibrium exists, it is unique.

Proof To prove this we need to show that the Beveridge curve is decreasing and the job creation curve is increasing in the (u, v) plane. For these, we need a couple of properties of $q_B(u, v)$ and $q_G(u, v)$ as they are defined in equation (42) and (43).

Lemma 8 *Some properties of $q_B(u, v)$.*

1. *$q_B(u, v)$ increasing in v,*
2. *$q_B(u, v)$ convex in u,*
3. *If $(-1+\gamma)\lambda - A(-2+\gamma)(-1+2\gamma) \leq 0$, then $q_B(u, v)$ is decreasing in u, otherwise it is increasing,*
4. *$uq_B(u, v)$ is increasing in u,*
5. *$\frac{uq_B(u,v)}{v}$ is decreasing in v if $(-1+\eta)\lambda + A(-1+2\gamma) < 0$.*

Proof 1.

$$\frac{\partial q_B(u, v)}{\partial v} = \frac{A\eta\lambda^2(2+\gamma(-1+u)-u)v^{-1+\eta}}{(\lambda - A(-1+2\gamma)uv^{\eta})^2} \tag{29}$$

The derivative is positive since $(2+\gamma(-1+u)-u) = 1-\gamma+1-u(1-\gamma) = (1-u)(1-\gamma)+1 > 0$.

2-3.

$$\frac{\partial q_B(u,v)}{\partial u} = \frac{A\lambda v^\eta\left((-1+\gamma)\lambda - A(-2+\gamma)(-1+2\gamma)v^\eta\right)}{(\lambda - A(-1+2\gamma)uv^\eta)^2} \tag{30}$$

The derivative is clearly increasing in u, it is negative if $(-1+\gamma)\lambda - A(-2+\gamma)(-1+2\gamma)v^\eta \le 0$, where the second term is positive.

4.

$$\frac{\partial u q_B(u,v)}{\partial u} = -\frac{A\lambda v^\eta\left(\lambda(-2+\gamma+2u-2\gamma u) + A(-1+\gamma)(-1+2\gamma)u^2v^\eta\right)}{(\lambda - A(-1+2\gamma)uv^\eta)^2} \tag{31}$$

The first term of the sum in the denominator is negative: $(-2+\gamma+2(1-\gamma)u)$ where $-2+\gamma$ is negative while the last term is positive. We may substitute $u = 1$ to the equation since if the sign is negative at $u = 1$, it is always negative. We get $-2+\gamma-2\gamma+2 = -\gamma$ which is negative. The other term of the sum is negative as well, since $\gamma - 1 < 0$ while $2\gamma - 1 \ge 0$. Hence, in total the derivative is positive.

5.

$$\frac{\partial \frac{uq_B(u,v)}{v}}{\partial v} = \frac{A\lambda(2+\gamma(-1+u)-u)uv^{-2+\eta}\left((-1+\eta)\lambda + A(-1+2\gamma)uv^\eta\right)}{(\lambda - A(-1+2\gamma)uv^\eta)^2} \tag{32}$$

$(2+\gamma(-1+u)-u)$ is positive as before (in the proof of lemma x.x), while the condition stated in the lemma makes the derivative negative for any u and v. ∎

Note that q_B has to be non-negative which requires that $\lambda - Av^\eta(2\gamma-1) \ge 0$, a sufficient condition is: $\lambda - A(2\gamma - 1) \ge 0$.

Lemma 9 *Some properties of $P(u)$:*

1. *$P(u)$ is decreasing in u,*
2. *$P(0) = 1$, $P(1) = 0$,*
3. *$P_u(1) = -1$, $P_u(0) = -0.5k(1+k)$,*
4. *$2P_u + uP_{uu} \le 0$.*

Proof 1. $P(u)$ can be rewritten as:

$$P(u) = (1-u)[1 + (1-u) + \cdots + (1-u)^{k_P-1}]$$

which is clearly decreasing in u.

3.

$$P_u \equiv \frac{\partial P}{\partial u} = \frac{-1 + (1-u)^k(1+ku)}{u^2}$$

which is clearly -1 at $u = 1$.

$$\lim_{u\to 0} P_u = \lim_{u\to 0} \frac{-k(1+k)(1-u)^{-1+k}u}{2u} = \lim_{u\to 0} \frac{k(1+k)(1-u)^{-2+k}(-1+ku)}{2} = \frac{-k(1+k)}{2}$$

4.

$$2P_u + uP_{uu} = 2\frac{-1+(1-u)^k(1+ku)}{u^2} + u\frac{2(-1+u)+(1-u)^k(2+(-1+k)u(2+ku))}{(-1+u)u^3} =$$
$$\frac{(1-u)^k(2(1+ku)(u-1)+2+(-1+k)u(2+ku))}{(-1+u)u^2} =$$
$$\frac{(1-u)^k(2u-2+2ku^2-2ku+2-2u-ku^2+ku2+k^2u^2)}{(-1+u)u^2} = -k(1+k)(1-u)^{k-1} \leq 0$$

∎

Lemma 10 *Some properties of* $q_G(u, v)$.

1. $q_G(u, v)$ *is increasing in* v,
2. $\frac{q_G(u,v)}{v}$ *is decreasing in* v *if* $(\eta - 1)\lambda + A(2\gamma - 1 + P(u^*))u^* < 0$ *where* $u^* = 1 - \sqrt[k_P]{\frac{2(1-\gamma)}{1+k_P}}$,
3. $q_G(u, v)$ *is concave in* u. *If* $\lambda(1-\gamma) \geq A2\gamma(1+\theta-\gamma)$, *then it is decreasing in* u *for any* $u \in (0, 1]$. *Otherwise, there exists a* $\bar{u}$ *such that* $q_G(u, v)$ *is increasing in* u *for* $\forall u \leq \bar{u}$, *and decreasing in* u *for* $\forall u > \bar{u}$.
4. *If* $\gamma\lambda \geq A(1 - (1-\gamma)(2\gamma - 1))$, $uq_G(u, v)$ *is increasing in* u.

Proof 1.

$$\frac{\partial q_G(u,v)}{\partial v} = \frac{A\eta\lambda^2(1+\theta+\gamma(-1+u)-u)v^{-1+\eta}}{(\lambda - A(-1+2\gamma+P)uv^\eta)^2} \tag{33}$$

where $(1 + \theta + \gamma(-1 + u) - u)$ is positive as before.

2.

$$\frac{\partial \frac{q_G(u,v)}{v}}{\partial v} = \frac{A\lambda(1+\theta+\gamma(-1+u)-u)v^{-2+\eta}\,((-1+\eta)\lambda + A(-1+2\gamma+P)uv^\eta)}{(\lambda - A(-1+2\gamma+P)uv^\eta)^2} \tag{34}$$

The derivative is negative iff $(-1+\eta)\lambda + A(-1+2\gamma+P)uv^\eta$ is negative. The condition stated in the Lemma says that the first negative term is higher in absolute value than the second positive term in the worst possible case: when $v = 1$ and $f(u) \equiv (-1 + 2\gamma + P(u))u$ takes it's maximum. Note that $f'(u) = -2 + 2\gamma + (1 + k)(1 - u)^k$; $f'(0) = k - 1 + 2\gamma > 0$; $f'(1) = 2(\gamma - 1) < 0$, while $f''(u) < 0$. Hence $f(u)$ has a maximum, which we obtain at $u^* = 1 - \sqrt[k_P]{\frac{2(1-\gamma)}{1+k_P}}$.

3.

$$\frac{\partial q_G(u,v)}{\partial u} = \frac{A\lambda v^\eta \left[(-1+\gamma)\lambda - Av^\eta((-1-\theta+\gamma)(-1+2\gamma+P) + u(-1-\theta+\gamma+u-\gamma u)P_u)\right]}{(\lambda - A(-1+2\gamma+P)uv^\eta)^2} \tag{35}$$

The sign of the derivative depends on the sign of the expression:

$$g(u) \equiv (-1+\gamma)\lambda - Av^\eta((-1-\theta+\gamma)(-1+2\gamma+P) + u(-1-\theta+\gamma+u-\gamma u)P_u)$$

$g(0) = \lambda(\gamma - 1) + Av^\eta(1 + \theta - \gamma)2\gamma$ where the first term is negative, the second positive. Hence we might have negative derivative here if the absolute value of the first term is higher than the second.

$g(1) = \lambda(\gamma - 1) + Av^\eta[(1 + \theta - \gamma)(2\gamma - 1) - \theta]$. First, we can see that this term is increasing in γ.

$$\frac{\partial g(1)}{\partial \gamma} = \lambda + Av^\eta(-(2\gamma - 1) + 2(1 + \theta - \gamma)) = \lambda + Av^\eta(-4\gamma + 3 + 2\theta)$$

which is positive since $\gamma \leq 1$. Hence we may evaluate $g(1)$ at $\gamma = 0$: $-\lambda - Av^\eta(1 + \theta)$ which is negative and evaluate it at $\gamma = 1$ is zero. Hence for any value of γ, $g(1) \leq 0$.

Further, we have that $g(u)$ is convex:

$$g'(u) = A^2\lambda(1+\theta+\gamma(-1+u)-u)v^{2\eta}(2P_u+uP_{uu}) = A^2\lambda(2+\gamma(-1+u)-u)v^{2\eta}[-k(1+k)(1-u)^{-1+k}] < 0$$

this derivative is negative since the last term is negative and all the previous terms are positive. Since $g(u)$ decreases monotonically wrt u we have that the derivative of $q_G(u, v)$ wrt u changes sign at most once.

If $g(0) \leq 0$, then $q_G(u, v)$ decreases in u everywhere on the interval $[0, 1]$. A sufficient condition for that is: $\lambda(\gamma - 1) + A(1 + \theta - \gamma)2\gamma < 0$.

If this condition does not hold, $g'(0) > 0$, $q_G(u, v)$ increases in u at $u = 0$. However, we know that at $u = 1$ the derivative of $q_G(u, v)$ wrt u is negative. So by the monotonicity of $g(u)$, there exists a $\bar{u}$ where the derivative is equal to zero.

4.

$$\frac{\partial u q_G(u,v)}{\partial u} = \frac{\left(A\lambda v^\eta \left(\lambda(1+\theta-\gamma+2(-1+\gamma)u) + Au^2v^\eta(-(-1+\gamma)(-1+2\gamma+P) + (1+\theta+\gamma(-1+u)-u)P_u\right)\right.}{(\lambda - A(-1+2\gamma+P)uv^\eta)^2} \tag{36}$$

Note that the sign of this derivative depends on the sign of $f(u) \equiv \lambda(1+\theta-\gamma+2(-1+\gamma)u) + Au^2v^\eta((1-\gamma)(-1+2\gamma+P)+(1+\theta+\gamma(-1+u)-u)P_u)$ where only the last term is negative because of P_u being negative.

First we may show that this function is decreasing in u:

$$f'(u) = 2(-1+\gamma)(\lambda - A(-1+2\gamma+P)uv^\eta) + A(1+\theta+\gamma(-1+u)-u)uv^\eta(2P_u+uP_{uu}) \leq 0$$

$\gamma-1$ is negative and it is multiplied by a positive number (the denominator of $q_G(u,v)$). The second term is negative as well since $2P_U + uP_{UU}$ is negative.

Since $f(u)$ is decreasing in u, if it is positive at $u=1$, it is always positive.

$$f(1) = \lambda(\theta-1+\gamma) + Au^2v^\eta((1-\gamma)(2\gamma-1)-\theta) = \theta(\lambda - Au^2v^\eta) + (1-\gamma)(Au^2v^\eta(2\gamma-1)-\lambda)$$

where the first term is positive as long as $\lambda > A$, in which case the second term is negative (since $2\gamma-1<1$). This also implies that $f(1)$ is increasing in θ, so it is always positive if it is for $\theta=1$. Evaluating $f(1)$ at $\theta=1$, we get that a sufficient condition for being positive is $\gamma\lambda \geq A(1-(1-\gamma)(2\gamma-1))$. This also implies that $\lambda > A$, since $(1-(1-\gamma)(2\gamma-1)) \geq \gamma \Leftrightarrow 2(1-\gamma)^2 \geq 0$.

We go back to the system of equations and try to eliminate e_B from the other equations. One equation defining the equilibrium is the sum of (20) and (21), the movement into and out of unemployment should be equal:

$$u(q_G(u,v)+q_B(u,v)) = \lambda(1-u) \tag{37}$$

e_B does not appear here. q_B and q_G are increasing in v, uq_B is increasing in u. Hence, under the condition that uq_G is increasing in u, this equation describes a negatively slopped Beveridge curve in the (u,v) plane.

We turn to the Job Creation curve and we try to show that it is positively slopped in the (u,v) plane.

The job creation curve is:

$$c = \frac{uq_B(u,v)(p-w_B)}{v(\delta+\lambda)} + \frac{uq_G(u,v)(\bar{p}-w_G)}{v(\delta+\lambda)} \tag{38}$$

where the wages are defined as in (23) and (24).

First, if we assume that $\lambda(\gamma-1)+A(2-\gamma)(2\gamma-1)<0$, then both q_B and q_G are decreasing in u. In this case if uq_G is increasing is u, we have that JC is positively slopped in the (u,v) plane: increase u, uq_G and uq_B increases while q_G and q_B decreases and with that the wages as well. Then under the condition that q_Bu/v and q_Gu/v are decreasing in v (what we assume in the proposition), we have to increase v as well to be in equilibrium.

Second, when $\lambda(\gamma-1)+A(2-\gamma)(2\gamma-1)\geq 0$, the problem is that q_B is increasing in u while q_G is increasing as well, at least for small values of u. Now we prove that in this case we still have JC positively slopped. The left-hand side is constant, the derivative of the rhs is the following:

$$\begin{aligned}\frac{\partial uq_B}{\partial u}(p-w_B)-uq_B\left(\frac{\partial w_B}{\partial q_B}\frac{\partial q_B}{\partial u}+\frac{\partial w_B}{\partial q_G}\frac{\partial q_G}{\partial u}\right)+\frac{\partial uq_G}{\partial u}(\bar{p}-w_G)-uq_G\left(\frac{\partial w_G}{\partial q_B}\frac{\partial q_B}{\partial u}+\frac{\partial w_G}{\partial q_G}\frac{\partial q_G}{\partial u}\right)= \\ \frac{\partial uq_B}{\partial u}(p-w_B)+\frac{\partial uq_G}{\partial u}(\bar{p}-w_G)-\frac{\partial w}{\partial q_B}\frac{\partial q_B}{\partial u}(uq_B+uq_G)-\frac{\partial w}{\partial q_G}\frac{\partial q_G}{\partial u}(uq_B+uq_G)\end{aligned} \quad (39)$$

where we used that the derivative of the two types of wages w_B and w_G is the same wrt to q_B and q_G, respectively. We can see that this derivative is positive as long as $\frac{\partial q_G}{\partial u}$ is negative and it's absolute value is higher than the derivative $\frac{\partial q_B}{\partial u}$. This is because the first two terms in the sum are positive ($p>w_B$, $\bar{p}>w_G$) and $\frac{\partial w}{\partial q_G}>\frac{\partial w}{\partial q_B}$:

$$\frac{\partial w}{\partial q_G}-\frac{\partial w}{\partial q_B}=\frac{(1-\beta)(\bar{p}-p)}{\delta+\lambda+\beta(q_B+q_G)}>0$$

In this case, the last term in the sum is positive and is higher then the absolute value of the preceding negative term.

We may also see that the derivative of the right-hand side of the JC curve is positive as well in the case when $\frac{\partial q_G}{\partial u}$ is positive or negative but has smaller absolute value than $\frac{\partial q_B}{\partial u}$.

To this end, we write $\frac{\partial uq_B}{\partial u}$ as $u\frac{\partial q_B}{\partial u}+q_B$ and the same for q_G and we take common factor:

$$\begin{aligned}u\frac{\partial q_G}{\partial u}\left(\bar{p}-w_G-\frac{\partial w}{\partial q_G}(q_B+q_G)\right)+q_G(\bar{p}-w_G)+ \\ u\frac{\partial q_B}{\partial u}\left(p-w_B-\frac{\partial w}{\partial q_B}(q_B+q_G)\right)+q_B(p-w_B)\end{aligned} \quad (40)$$

where

$$\bar{p}-w_G-\frac{\partial w}{\partial q_G}(q_B+q_G)=\frac{(-1+\beta)(\delta+\lambda)((b-\bar{p})(\delta+\lambda)+(p-\bar{p})q_B)}{(\delta+\lambda+\beta(q_B+q_G))^2}>0$$

this also implies that $p-w_B-\frac{\partial w}{\partial q_B}(q_B+q_G)>0$ since this term is higher than the previous one. This latter is because $p-w_B=\bar{p}-w_G$ (see the section on wages: $w_G-w_B=\bar{p}-p$) and we also have that the derivative of the wage wrt q_G is higher than wrt q_B (as shown above).

Hence, if $\frac{\partial q_G}{\partial u}$ is positive, we are done. If it is negative but has lower absolute value than $\frac{\partial uq_B}{\partial u}$, we are also done, since this latter has a higher multiplier.

So we have shown that the right-hand side of JC has a positive derivative wrt u. To get equilibrium after an increase in u, we need to increase v as well. This is under the condition that $\frac{uq_B}{v}$ and $\frac{uq_G}{v}$ are decreasing in v. Note that the wages increase after an increase of v, since q_B and q_G increase.

Therefore, BC describes a monotone negative relationship in (u, v) while JC describes a positive one. Hence, they might cross only once, the equilibrium is unique if exists.

We sum up the conditions we have made to get this result. $q_B u/v$ has to be decreasing in v: $(-1+\eta)\lambda + A(-1+2\gamma) < 0$. Note that this implies that $q_G u/v$ is decreasing in v as well. The condition for this latter was $(-1+\eta)\lambda + A(2\gamma - 1 + P(u))u < 0$ at the maximum u^*. We evaluate $(-1+2\gamma+P(u))u$ at the maximum, we get:

$$-1+2\gamma-\left(\frac{2-2\gamma}{1+k_P}\right)^{\frac{1}{k_P}}\left(-2+2\gamma+\left(\left(\frac{2-2\gamma}{1+k_P}\right)^{\frac{1}{k_P}}\right)^{k_P}\right) = -1+2\gamma-\left(\frac{2-2\gamma}{1+k_P}\right)^{\frac{1}{k_P}}\left(\frac{(2\gamma-2)k_P}{1+k_P}\right)$$

which is smaller that $2\gamma - 1$, hence in this case $\lambda(\eta - 1)$ needs to be "less negative".

Further, we assumed that uq_G is increasing in u, this happens if $\gamma\lambda \geq A(1-(1-\gamma)(2\gamma-1))$.

At last, we assumed that w_G and w_B are increasing in q_G: $\bar{p} - p \leq \frac{\beta(\bar{p}-b)}{\beta+\delta+\lambda}$ ∎

7.3.2 Proposition 2

Proof In this case the offer arrival probabilities are given by the following equations:

$$q_B(u, v, e_B, e_G) = Av^{\eta}(1 + \gamma e_B + (1-\gamma)e_G)$$

$$q_G(u, v, e_B, e_G) = Av^{\eta}(1 + \gamma e_G + (1-\gamma)e_B)$$

By dividing the two equilibrium conditions (20) and (21) with each other, we get that:

$$\frac{q_B}{q_G} = \frac{e_B}{e_G} = \frac{1+\gamma e_B + (1-\gamma)e_G}{1+\gamma e_G + (1-\gamma)e_B}$$

Rearranging this latter equality we obtain:

$$e_B(1+\gamma e_G + (1-\gamma)e_B) = e_G(1+\gamma e_B + (1-\gamma)e_G)$$

$$e_G - e_B + (1-\gamma)(e_G^2 - e_B^2) = 0$$

$$(e_G - e_B)(1 + (1-\gamma)(e_G + e_B)) = 0$$

since the second term is always positive, the multiplication can be zero only if $e_G = e_B$.

If $\theta = 1$, the market provides bad and good offers with probability 0.5 each. Substituting $e_B = e_G$ into the definition (3) implies that the family arrival puts equal weights on the two wages as well which are independent of the homophily index γ. By $e_B = e_G$, q_B and q_G do not depend on γ either. ∎

7.3.3 Proposition 3

Proof (i) First, we prove that if $\theta > 1$, $e_G > e_B$.

If $k_P = 0$ and $\theta > 1$, the ratio of bad and good employed in the equilibrium is given by

$$\frac{q_B}{q_G} = \frac{e_B}{e_G} = \frac{1 + \gamma e_B + (1-\gamma)e_G}{\theta + \gamma e_G + (1-\gamma)e_B}$$

After rearranging:

$$\theta e_B - e_G = (1-\gamma)(e_G^2 - e_B^2) \tag{41}$$

Assume the contrary, $e_B \geq e_G$. This implies that the right-hand side of this expression is negative, then the left-hand side should be as well: $e_G \geq \theta e_B$. This can only be if $\theta \leq 1$, the contrary of our assumption that $\theta > 1$. Hence, $e_G > e_B$.

For any equilibrium where $e_G > e_B$, an increase in γ decreases q_B/q_G (nominator decreases, denominator increases). We can express θ from equation (41):

$$\theta = \frac{(1-\gamma)(e_G^2 - e_B^2)}{e_B} + \frac{e_G}{e_B}$$

If θ increases, the right-hand side has to increase as well, which is only consistent with an increase in $e_G - e_B$.

(ii)-(iii) To compare the expected wages of family and market, we compare the weight which is put on the high wage in the conditional expectation. For the market this is $\theta/(1+\theta)$ and for the family:

$$\frac{\gamma e_G + (1-\gamma)e_B}{e_B + e_G} = \frac{\gamma + (1-\gamma)\frac{e_B}{e_G}}{1 + \frac{e_B}{e_G}}$$

First of all, we may see that as γ increases, the market weight does not change, while the family weight increases as $e_G > e_B$, which is the case if $\theta > 1$. As γ changes, $x \equiv e_B/e_G < 1$ gets smaller, the derivative of the family weight wrt γ:

$$\frac{1 - x^2 + (1-2\gamma)\frac{\partial x}{\partial \gamma}}{(1+x)^2}$$

which is clearly positive as long as $\gamma > 0.5$. Hence with homophily the family expected wage improves.

Second, we show that for $\gamma = 1$ the market expectation is the same as the family expectation. We use the equlibrium condition (41) to express θ:

$$\theta = \frac{(1-\gamma)(e_G^2 - e_B^2)}{e_B} + \frac{e_G}{e_B}$$

which is equal to $\frac{e_G}{e_B}$ if $\gamma = 1$. This is equal to the family weight on the good wage at $\gamma = 1$. Since decreasing γ makes the family expectation worse while does not change the market one, we have the result. ∎

7.3.4 Proposition 4

Proof We can use (20) to express $e_B = \frac{q_B u}{\lambda}$. Substituting this identity into (18), we can find q_B as a function of u and v:

$$q_B(u,v) = \frac{\lambda A v^\eta [1 + (1-\gamma)(1-u)]}{\lambda - A v^\eta u(2\gamma - 1)} \tag{42}$$

We may do the same with respect to q_G, now using (21), we have $e_B = 1 - u - \frac{q_G u}{\lambda}$. We write (19) in the following way:

$$q_G(u,v,e_B) = A v^\eta [\theta + (1-u)(1-\gamma) - e_B(1 - 2\gamma - P)] \tag{43}$$

where $P(u) = (1-u)\frac{1-(1-u)^{k_P}}{u}$ and $P = 0$ if $k_P = 0$, $P > 0$ when $k_P > 0$.

Again, substituting e_B we get $q_G(u,v)$:

$$q_G(u,v) = \frac{\lambda A v^\eta [\theta + (1-u)(1-\gamma)]}{\lambda - u A v^\eta (2\gamma + P - 1)} \tag{44}$$

If $\theta = 1$, the nominator is the same in the two equations while the denominator differs only in the term P which makes $q_G > q_B$. Again dividing (20) and (21), we get that q_G/q_B=e_G/e_B:

$$\frac{e_G}{e_B} = \frac{q_G(u,v)}{q_B(u,v)} = \frac{\lambda - A(-1+2\gamma)uv^\eta}{\lambda + A(1-2\gamma-P)uv^\eta}$$

On the other hand, the number of the professional contacts enters only through P which is increasing in k_P:

$$\frac{\partial P(u)}{\partial k_P} = -\frac{(1-u)^{1+k}\log(1-u)}{u}$$

which is positive.

Turning to the derivative wrt the homophily parameter γ, notice that the derivative both of the nominator and the denominator wrt γ is equal to $-2Auv^\eta$. So, we can write the derivative of the fraction wrt γ:

$$\frac{\partial\frac{q_G}{q_B}}{\partial\gamma}=\frac{-2Auv^{\eta}(\lambda+A(1-2\gamma-P)uv^{\eta}-(\lambda-A(-1+2\gamma)uv^{\eta}))}{(\lambda+A(1-2\gamma-P)uv^{\eta})^2}=\frac{-2Auv^{\eta}(Auv^{\eta}(-P))}{(\lambda+A(1-2\gamma-P)uv^{\eta})^2}>0$$

This expression is clearly positive, hence the ratio e_G/e_B is increasing in γ.

Now, if $k_P=0$, $P=0$, the denominator is the same in the equation of q_B and q_G. Dividing them we obtain:

$$\frac{e_G}{e_B}=\frac{q_G}{q_B}=\frac{\lambda Av^{\eta}[\theta+(1-u)(1-\gamma)]}{\lambda Av^{\eta}[1+(1-\gamma)(1-u)]}$$

Obviously, if $\theta>1$, then $e_G/e_B>1$ and this ration increases in θ. Again, both the derivatives of the nominator and denomitor wrt γ are $-\lambda Av^{\eta}(1-u)$. Then,

$$\frac{\partial\frac{q_G}{q_B}}{\partial\gamma}=\frac{-\lambda Av^{\eta}(1-u)(-\lambda Av^{\eta}\theta)}{(\lambda Av^{\eta}[1+(1-\gamma)(1-u)])^2}>0$$

Hence, the difference e_G-e_B is increasing in γ. ∎

7.3.5 Proposition 5

Proof (i) Again, we compare the weight on the good wages between the wage expectation of markets and family contacts. For the market this is $\theta/(1+\theta)$, which does not change with γ. For the family we have:

$$\frac{\gamma e_G+(1-\gamma)e_B}{e_B+e_G}$$

This is decreasing in γ if $e_G>e_B$ as shown by the proof of Proposition 3.2.

To be able to compare the two quantities we divide the equilibrium conditions (20) and (21):

$$\frac{q_G}{q_B}=\frac{e_G}{e_B}=\frac{\theta+\gamma e_G+(1-\gamma)e_B+e_GP}{1+\gamma e_B+(1-\gamma)e_G}$$

where $P\equiv(1-u)\frac{1-(1-u)^{k_P}}{u}$

From here we express θ:

$$\theta=\frac{(1-\gamma)(e_G^2-e_B^2)+e_G}{e_B}-e_GP$$

And from here we can express the market weight on good wages:

$$\frac{\theta}{1+\theta}=\frac{(e_G^2-e_B^2)(1-\gamma)+e_G(1-e_BP)}{(e_G+e_B)(1+(e_G-e_B)(1-\gamma))-e_Ge_BP}$$

First we compare the two weights when $\gamma = 0$. In this case the market always performs better as long as $e_G > e_B$ and $\theta > 1$:

$$\frac{\theta}{1+\theta} > \frac{e_B}{e_G + e_B} \iff \frac{1}{\theta} < \frac{e_G}{e_B}$$

since the last inequality the left-hand side is smaller than 1 while the right-hand side is higher.

On the other hand, if $\gamma = 1$, the family expected wage is always higher. Evaluating the two weights at $\gamma = 1$, for the market:

$$\frac{\theta}{1+\theta} = \frac{e_G(1 - e_B P)}{e_B + e_G(1 - e_B P)}$$

for the family:

$$\frac{e_B}{e_B + e_G}$$

We define the function $f(y) = \frac{y}{y+e_B}$, which is clearly strictly increasing in y. Hence we have that $f(e_G) > f(e_G(1 - e_B P))$ since $1 - e_B P < 1$. That is the family weight is higher.

Since the market weight does not change in γ while the family weight is increasing, by continuity there is a threshold $\bar{\gamma}_1 \in (0, 1]$ where the two weights are equal and above which the family performs better.

(ii) By equalizing the two weights on the good wages we find the threshold $\bar{\gamma}_1$:

$$\bar{\gamma}_1 = \frac{\theta}{1+\theta}\frac{x+1}{1-x} - \frac{x}{1-x}$$

where $x = e_B/e_G$ which decreases with k_P. Hence what remains to show is that the threshold $\bar{\gamma}_1$ is increasing in x. The derivative is:

$$\frac{-1+\theta}{(1+\theta)(-1+x)^2} < 0$$

which is negative since $\theta > 1$.

(iii) As for the overall network (family + professional contacts), the weight on the good wage in the expectation is:

$$\frac{\gamma e_G + (1-\gamma)e_B + e_G P}{e_B + e_G(1+P)} = \frac{\gamma + (1-\gamma)x + P}{x + (1+P)}$$

where $x \equiv \frac{e_B}{e_G} < 1$ is decreasing in γ. Hence the weight is increasing in γ, the derivative is positive:

$$-\frac{(-1+x)(1+P+x) + (-1+\gamma(2+P))\frac{\partial x}{\partial \gamma}}{(1+P+x)^2} > 0$$

which is positive since $x < 1$ and $\gamma > 0.5$.

Now we show that for $\gamma = 1$, the network wage is higher than the market: $f(e_G(1 + P)) > f(e_G(1 - e_B P))$ where $f(y)$ is defined as before. As we decrease γ, the network expectation gets worse, but at $\gamma = \bar{\gamma}_1$ it is still higher than the family expectation:

$$\frac{\gamma e_G + (1-\gamma)e_B}{e_B + e_G} < \frac{\gamma e_G + (1-\gamma)e_B + e_G P}{e_B + e_G(1+P)}$$

which is true for any value of γ including $\bar{\gamma}_1$.

Hence, by continuity, for a $\gamma = \bar{\gamma}_1 - \epsilon$, the family is worse than the market but the overall network is still better. $\bar{\gamma}_2$ is the solution of the equation:

$$\frac{\theta}{\theta+1} = \frac{\bar{\gamma}_2 e_G + (1-\bar{\gamma}_2)e_B + e_G P}{e_B + e_G(1+P)}$$

If $\theta \geq 2$, $\bar{\gamma}_2$ is surely higher than zero. At $\gamma = 0$, the network weight is: $\frac{x+P}{1+x+P}$ which is at most 2/3 (since both P and x take their maximum at 1). So the market weight is higher if:

$$\frac{\theta}{1+\theta} \geq \frac{2}{3} \iff \theta \geq 2$$

(iv) Expressing the threshold $\bar{\gamma}_2$ from here, we get:

$$\bar{\gamma}_2 = \frac{\theta}{1+\theta}\frac{1+x+P}{1-x} - \frac{x+P}{1-x}$$

where x is defined as before. If the number of professinal contacts increase, x decreases and the threshold decreases as well if $\theta \geq 2$. The derivate of $\bar{\gamma}_2$ wrt x:

$$\frac{-1-P+\theta}{(1+\theta)(-1+x)^2} > 0$$

where P is at most 1. ∎

7.3.6 Proposition 6

Proof We compare the weight of good wages in the market expectation $(\theta/(1+\theta)$ with the weight in the total network expectation (as defined in Definition 4):

$$\frac{\theta}{1+\theta} \gtreqless \frac{\gamma e_G + (1-\gamma)e_B + e_G P}{e_B + e_G(1+P)} \iff \frac{1}{\theta}+1 \lesseqgtr \frac{e_B + e_G(1+P)}{\gamma e_G + (1-\gamma)e_B + e_G P} \iff \frac{1}{\theta} \lesseqgtr \frac{(1-\gamma)e_G + \gamma e_B}{\gamma e_G + (1-\gamma)e_B + e_G P}$$

where where $P \equiv (1-u)\frac{1-(1-u)^{kP}}{u}$ and we took inverse in the first transformation.

The right-hand side of the last comparison is increasing in e_G and decreasing in e_B, derivative wrt e_G:

$$\frac{e_B(-1+\gamma(2+P))}{(e_G + e_B\gamma - e_G\gamma)^2} > 0$$

which is surely positive if $\gamma > 0.5$. The derivate wrt e_B is just the same but multiplied by -1. The network expectation is the highest when the expression on the rhs is the lowest. This we obtain by evaluating it at $e_G = 0$ and $e_B = 1$. We also take $P = 1$, the highest possible value. Hence the market is better if:

$$\frac{1}{\theta} < \frac{\gamma}{1 - \gamma + 1} \Longleftrightarrow \theta > \frac{2 - \gamma}{\gamma}$$

∎

7.3.7 Proposition 7

Proof If k_P or θ increases, q_B does not change, while q_G increases for every value of u and v. The Beveridge Curve moves down in the (u, v) plane: the left-hand side of $u(q_B(u, v) + q_G(u, v)) = \lambda(1 - u)$ increases, to decrease it, we need to decrease v.

The job creation curve moves to the left. The derivative of the left-hand side of (22) with respect to k_P is the following:

$$-uq_B \frac{\partial w_B}{\partial q_G} \frac{\partial q_G}{\partial k_P} - uq_G \frac{\partial w_G}{\partial q_G} \frac{\partial q_G}{\partial k_P} + \frac{\partial q_G}{\partial k_P} u(\bar{p} - w_G) = u \frac{\partial q_G}{\partial k_P} \left(\bar{p} - w_G - (q_G + q_B) \frac{\partial w}{\partial q_G} \right)$$

where we used that the wages w_B and w_G have the same derivatives wrt q_G. The derivate wrt to θ is just the same. This expression is positive since the term in the parenthesis is positive (see the proof of Proposition 2.1 in the Appendix).

Hence if k_P or θ increases the right-hand side of JC increases, to get equilibrium we need to increase v for every value of u. In this case, uq_B/v and uq_G/v decrease while wages will increase under the conditions of Proposition 2.1. ∎

7.4 Tables

γ/k_P	1	2	3	4	5	10	15	20	25	30
0.6	-0.092	-0.092	-0.091	-0.091	-0.091	-0.089	-0.087	-0.086	-0.085	-0.084
0.7	-0.092	-0.092	-0.091	-0.091	-0.090	-0.089	-0.087	-0.086	-0.085	-0.084
0.8	-0.092	-0.092	-0.091	-0.091	-0.090	-0.088	-0.087	-0.086	-0.085	-0.084
0.9	-0.092	-0.092	-0.091	-0.091	-0.090	-0.088	-0.087	-0.086	-0.085	-0.084
0.95	-0.092	-0.092	-0.091	-0.091	-0.090	-0.088	-0.087	-0.086	-0.085	-0.084
1	-0.092	-0.092	-0.091	-0.091	-0.090	-0.088	-0.087	-0.086	-0.085	-0.084

Table 5: Optimality of unemployed decision: the value of $q_G(W_G - U) - \lambda(U - W_B) + b - w_B$ in equilibrium. This should be negative if the decision is optimal.