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Saving in an Aging Society with Public Pensions: A Model with Decades *

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Abstract

We consider a quite realistic model of an economy with an aging population and consider the intergenerational equity of pension systems and their reforms. Filling the model with numbers, we are able to compare different policies: 1. the basic run, 2. the reduced accrual rates, 3. replacing wage indexation with price indexation and 4. raised retirement age. Whether the policy changes are anticipated or not, the private reactions widely differ.

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1. Introduction

There is a growing practical and theoretical interest toward the implication of an aging population for intergenerational equity, especially concerning pension systems. The papers mentioned in the reference list modeled this issue in one way or another.... To construct a different, and from some point of view, more realistic description of this complex process than these papers did, in this paper we shall build up a parsimonious pension model with the following characteristics: (a) The aging process is exogenous, broken down to periods rather than two or three overlapping generations and is driven by dropping fertility and increasing longevity. (b) There is a public pension system with changing contribution and accrual rates. (c) The growth rate of the individual total wage cost is exogenous and time-invariant. (d) The ratio of the interest factor to the growth factor of the wage bill is time-invariant. To derive asset dynamics as well, we add optimizing households. (e) The household's consumption and the derived utility depend on the size of the household (Blundell et al. 1994). (f) The households may have various (rational or naive) expectations concerning pension policy.

Major results of the paper are as follows. 1. The base run reflects the properties of an economy with a steeply rising endogenous determined contribution rate. The corresponding consumption and asset paths describe a fast transition from a stationary population into a contracting stable population. 2. In a variant, we consider a pension policy which at a given date, drastically reduces the accrual rates, i.e. pensions. 3. In a second variant, wage-indexed pensions are replaced by price-indexed pensions from a given date. 4. In another variant, we analyze a delayed rise in the retirement age, drastically reducing the pension burden. The modified endogenous paths reflect the necessary adjustments and the changes in the burden of subsequent cohorts.

The structure of the reminder of the paper is as follows: Section 2 outlines a parsimonious macromodel with given consumption paths. Section 3 introduces optimal consumption paths to derive asset dynamics. Section 4 displays the numerical calculations. Section 5 concludes. The Appendix contains the more sophisticated proofs.

Further scenarios will be discussed in future versions: privatization of part of the public system, etc.

2. A macromodel

We start the presentation with a demographic block and continue with a pension block, yielding the macroblock.

Demographic block

Here we describe the demographic block which is simple enough but works with periods rather than generations and takes care of dropping fertility and increasing life expectancy.

Let t be the index of calendar periods, $t = \ldots, -1, 0, 1, \ldots$ but also $\ldots, 1940, 1950, 1960, \ldots$. We shall always use the following principle of notation: when a quantity depends on age as well as on calendar time, then the first index refers to age, and the second to the calendar time. Let us denote the number of aged i in period t by $n_{i,t}$. For simplicity, we assume that every member of a cohort dies at the same age, i.e. there is no longevity risk.

Concerning changing life expectancy, we must differentiate between life expectancy of the *deceased* in period t: I_t and the life expectancy of those *born* in period t: $\mathbf{I}_t$. (The traditional term is misleading, since it expresses the former while refers to the latter.) We have the following relation between them: $\mathbf{I}_{t-I_t} = I_t$. For example, if the expected age of people at death is equal to 80 in 2050, then $\mathbf{I}_{1970} = I_{2050}$. $\mathbf{I}_t$ appears in the longitudinal, individual level, while I_t appears in the macrorelations, determining cross-sectional balances.

To get rid of the complexities of a two-sex world, we assume half households. If the parent was born in period t, then all his children are born at once, in period $t + H$, and their number is $2f_{t+H}$, f_{t+H} staying with him/her, the other f_{t+H} with her/him, where H is a positive integer. A child stays with his parent until age L, when he starts to work, L also being a positive integer. People born in t retire at age $\mathbf{J}_t$, gnerally a time-varying integer. Again, $J_t = \mathbf{J}_{t-\mathbf{J}_t}$. We assume that only workers have children in their household, i.e. $L < H < \mathbf{J}_t - L$.

To sum up, a person born in period t, starts working in period $t + L$, gives birth to children in period $t + H$, separates from them in period $t + H + L$, retires in period $t + \mathbf{J}_t$ and dies in period $t + \mathbf{I}_t$.

We have the following demographic equations for $t \geq 0$:

$$n_{i,t} = \begin{cases} f_t n_{H,t} & \text{if } i = 0; \\ n_{i-1,t-1} & \text{if } i = 1, \ldots, \mathbf{I}_t; \\ 0 & \text{if } i > \mathbf{I}_t. \end{cases}$$

In the transition of dynamic systems, we assume that the initial values of birth numbers, namely $n_{0,-I_0}, n_{0,-I_0+1}, \ldots, n_{0,-1}$ are given.

Denote $N_t = \sum_{i=0}^{I_t}$ the population size in period t, then growth factor of the total population is equal to

$$\nu_t = \frac{N_t}{N_{t-1}}.$$

Assuming that the *relative interest factor* α is constant, the endogenous interest factor is the product of the relative interest factor and the growth factor of the total productivity:

$$R_t = \alpha \nu_t g.$$

Note that this is a short-cut. In the general equilibrium theory, the interest factor is determined from the macroequilibrium conditions: either assuming rational expectations concerning the interest rate (e.g. Auerbach and Kotlikoff, 1987) or naive expectations (cf. Molnár and Simonovits, 1998).

In steady states, ω can be expressed as $g\nu$, thus $R = \alpha \nu g$.

Let us denote $w_{i,t}$ the household head's total wage at age i in period t. We assume that as time passes, the earning–age function is multiplied by the time-invariant productivity growth factor $g > 1$:

$$w_{i,t} = w_{i,L} g^{t-L} = w_i g^{t-L}, \qquad i = L+1, L+2, \ldots, \mathbf{J}_t \quad \text{and} \quad t = -2, -1, 0, 1, 2, \ldots,$$

where w_i is the wage-profile in period L and $w_{L,L} = 1$. (Note that the wage structure may depend on the demographic situation, as is persuasively argued by Akihiko (2006, Figure 35, p. 143), but we neglect this fact.) Later on, it will be useful to define $w_{i,t+i} = 0$ for $i > \mathbf{J}_t$.

First we define the aggregate total wage:

$$W_t = \sum_{i=L}^{J_t} n_{i,t} w_{i,t}.$$

Pension block

Here we outline our pension block, first a pay-as-you-go *public pension system* and explain income $y_{i,t}$ from wage $w_{i,t}$ and pension benefit $b_{i,t}$. We have already met retirement age $\mathbf{J}_t$ of the person born in period t (an integer) and the retirement age in period $t + \mathbf{J}_t$, determined by the government. We can now describe the impact of the pension system as follows. Our individual, born in t, contributes $\tau_{i,t+i} w_{i,t+i}$ to the public pension system at age $i = L, L+1, \ldots, \mathbf{J}_t$ and receives a *pension benefit* $b_{i,t+i}$ at age $i = \mathbf{J}_t + 1, \ldots, \mathbf{I}_t$. Thus his income path changes as follows:

$$y_{i,t+i} = \begin{cases} (1 - \tau_{t+i}) w_i g^{t-L+i} & \text{if } L \le i \le \mathbf{J}_t; \\ b_{i,t+i} & \text{if } \mathbf{J}_t \le i \le \mathbf{I}_t. \end{cases}$$

We assume a pension system, where the *entry benefit* is given by as a linear function of past net wages, the coefficient being called *accrual rates*:

$$b_{\mathbf{J}_t+1,t+\mathbf{J}_t+1} = \sum_{j=L}^{\mathbf{J}_t} \theta_{t+j} (1 - \tau_{j,t+j}) w_{j,t+j} g^{\mathbf{J}_t - j + 1},$$

i.e.

$$b_{\mathbf{J}_t+1,t+\mathbf{J}_t+1} = \sum_{j=L}^{\mathbf{J}_t} \theta_{t+j} (1 - \tau_{j,t+j}) w_{j,0} g^{t-L+\mathbf{J}_t+1}.$$

Note that in practice, θ_{t+j} and $\tau_{j,t+j}$ may change in time, but in theory, if we assume time-invariant accrual rate θ and contribution rate τ, then the entry benefit is proportional to *valorized lifetime net earnings* $(1 - \tau)\hat{w}_{t+\mathbf{J}_t+1}$:

$$b_{\mathbf{J}_t+1,t+\mathbf{J}_t+1} = \theta(1 - \tau)\hat{w}_{t+\mathbf{J}_t+1},$$

where

$$\hat{w}_{t+\mathbf{J}_t+1} = \sum_{j=L}^{\mathbf{J}_t} w_{j,t+j} g^{\mathbf{J}_t - j + 1} = g^{t-L+\mathbf{J}_t+1} \sum_{j=L}^{\mathbf{J}_t} w_{j,L}.$$

The *continued benefits* are wage-price-indexed with shares ι_t and $1 - \iota_t$, respectively:

$$b_{i+1,t+i+1} = b_{i,t+i} g^{\iota_t}, \qquad i = \mathbf{J}_t + 1, \ldots, \mathbf{I}_t - 1.$$

We consider the individual *pension wealth* which plays a prominent role in the evaluation of the burden of unfunded pension systems. We shall define the implicit pension wealth $d_{i,t+i}$ of a person at age i as the present value of outstanding pension benefits at the end of this period. To formulate the implicit pension debt, we need to introduce the *compounded interest factor* in the time interval $[v, z]$:

$$\rho_{v,z} = \prod_{t=v+1}^{z} R_t \quad \text{for} \quad z > v \quad \text{and} \quad \rho_{v,v} = 1.$$

For a worker, who was born in period t, his earning $w_{h,t+h}$ in period $t+h$ will yield pension "part" $\theta_{t+h} w_{h,t+h} g^{i-h}$ in period $t+i$, for $h = L, \ldots, \mathbf{J}_t$ and $i = \mathbf{J}_t + 1, \ldots, \mathbf{I}_t$. Taking account of $w_{h,t+h} = w_{h,L} g^{t+h-L}$, summing up, and discounting to the period $t+j$ yields *pension wealth*

$$d_{j,t+j} = g^{t-L} \sum_{h=L}^{j} \theta_{t+h}(1 - \tau_{h,t+h}) w_{h,L} \sum_{i=\mathbf{J}_t+1}^{\mathbf{I}_t} g^i \rho_{t+j,t+i}^{-1}, \qquad j = L, \ldots, \mathbf{J}_t.$$

For a pensioner, the sum of the remaining claims between periods $t+i+1$ and $t+\mathbf{I}_t$:

$$d_{i,t+i} = \sum_{h=i+1}^{\mathbf{I}_t} b_{h,t+h} \rho_{t+i,t+h}^{-1}, \qquad i = \mathbf{J}_t + 1, \ldots, \mathbf{I}_t.$$

In aggregate relations, profiles rather than paths appear. (Pension wealth profiles are derived in the Appendix.) The pension system need not be balanced, i.e. the difference between the aggregate benefits and the aggregate contribution defines the *pension budget deficit*:

$$G_t = \sum_{i=L}^{I_t} n_{i,t}(b_{i,t} - \tau_t w_{i,t}).$$

The *explicit* debt of the public pension system follows the well-known dynamic equation:

$$D_t^E = R_t D_{t-1}^E + G_t.$$

Aggregating the individual implicit pension wealths, the aggregate *implicit* pension debt is as follows:

$$D_t^I = \sum_{i=L}^{I_t} n_{i,t} d_{i,t}.$$

Adding together the implicit and explicit pension wealths, results in the *total pension wealth*

$$D_t = D_t^I + D_t^E.$$

The pension system is called *intergenerationally equitable* if the total pension debt grows parallel with the output: D_t/Y_t is constant.

In a lot of countries in a lot of periods, pension systems are purely pay-as-you-go, i.e. the annual pension benefit is equal to zero: $G_t = 0$. Then the contribution rate $\tau_t^{\rm o}$ is equal to

$$\tau_t^{\rm o} = \frac{\sum_{i=J_t+1}^{I_t} n_{i,t} b_{i,t}}{\sum_{j=L}^{J_t} n_{j,t} w_{j,t}}.$$

But $b_{i,t}$ depends on τ_v, $v < t$ as seen above.

In a general model, we have as initial conditions $(b_{i,t})$ and then $\tau_t^{\rm o}$ is determined. Note, however, that $(b_{i,t})$ depend in turn on $\tau_{-J_1+t}, \ldots, \tau_{-1+t}$. If we do not want to define initial contribution rates, we may assume that the system started in a steady state. Then with substitution, and assuming wage-indexed pensions with $\iota_t = 1$: $b_{i,t} = b_t$ and denoting the number of pensioners by P_t yields a steady-state equation and contribution rate:

$$\tau W_t = \theta P_t \hat{w}_t - \tau_t \theta P_t \hat{w}_t, \quad \text{i.e.} \quad \tau_t = \frac{\theta P_t \hat{w}_t}{W_t + \theta P_t \hat{w}_t}, \qquad t < 0.$$

Until now we have taken consumption paths as given, from now we can derive them form individual life-cycle optimization.

3. Optimal consumption paths

In this section first we discuss a simple life cycle model where households maximize a standard utility function under a standard budget constraint. Second, we introduce complications like habit formation, inheritance and credit constraint and shocks. This helps us to derive asset dynamics, as well as considering the partial prefunding of the unfunded pension system.

A simple household life cycle

As is usual, we build up the macro block from microeconomic variables. Let us denote $c_{i,t}$ the household head's consumption at age i in period t. At this stage, we take the adult consumption as given and delay its explanation for a while.

Consider an adult born in period t and his household in later periods. We assume that each child consumes μ times of his parent's $(0 < \mu \leq 1)$. Let us denote the household consumption size by $m_{i,t}$ (cf. Meier and Wrede, 2005). Then we have

$$m_{i,t+i} = \begin{cases} 1 + \mu f_{t+H} & \text{if } H \leq i < H + L; \\ 1 & \text{if } L \leq i < H \text{ or } H + L \leq i \leq \mathbf{I}_t. \end{cases}$$

Here we introduce aggregate consumption in period t:

$$C_t = \sum_{i=L}^{I_t} n_{i,t} m_{i,t} c_{i,t}.$$

Following Krueger (2004), at the-end-of-period accumulated assets $a_{i,t+i}$ and per-period saving $s_{i,t+i}$ of a household are defined as

$$a_{i,t+i} = R_{t+i} a_{i-1,t-1+i} + y_{i,t+i} - m_{i,t+i}\ c_{i,t+i}$$

and

$$s_{i,t+i} = a_{i,t+i} - a_{i-1,t-1+i} = a_{i-1,t-1}(R_{t+i} - 1) + y_{i,t+i} - m_{i,t+i}c_{i,t+i},$$

respectively. The initial and end values are equal to zero: $a_{-1,t} = 0 = a_{I_t,t}$

We define aggregate assets and saving respectively as

$$A_t = \sum_{i=L}^{I_t} n_{i,t}a_{i,t} \quad \text{and} \quad S_t = \sum_{i=L}^{I_t} n_{i,t}s_{i,t}.$$

By definition, $A_t = A_{t-1} + S_t$.

Since working households pay pension contributions and pensioner households receive pension benefits, their income $y_{i,t+i}$ differs from their earning $w_{i,t+i}$. Therefore, we shall formulate the budget constraint with the former rather than the latter. Using the capital (present) values of incomes and consumption in period $t + L$, the lifetime budget constraint of the person born in period t is as follows:

$$\sum_{i=L}^{\mathbf{I}_t} \rho_{t+L,t+i}^{-1}(y_{i,t+i} - m_{i,t+i}\ c_{i,t+i}) = 0.$$

To determine the optimal consumption path $(c_{i,t+i})$, we assume the following household lifetime utility function:

$$\sum_{i=L}^{\mathbf{I}_t} \delta^{i-L}u_i(c_{i,t+i}),$$

where $0 < \delta \leq 1$ is the discount factor and $u_i(c_{i,t+i})$ is the household head's per-period utility function at age i. To take care of the age-specific and time-variant household size, we assume that the per-period utility function at age i is equal to a time-invariant per-capita utility function multiplied by the size of the household and indicator variable $\beta_{i,t+i}$, equaling to 1 if the consumer works and dropping to $0 < \beta < 1$ if he is retired (cf. Scholz et al., 2006):

$$u_i(c_{i,t+i}) = \beta_{i,t+i}\ m_{i,t+i}\ u(c_{i,t+i}),$$

where

$$\beta_{i,t+i} = \begin{cases} 1 & \text{if } L \leq i \leq \mathbf{J}_t; \\ \beta & \text{if } \mathbf{J}_t < i \leq \mathbf{I}_t. \end{cases}$$

To obtain nice analytical results, we must assume a CRRA-utility function:

$$u(x) = \begin{cases} \dfrac{x^{1-\gamma}}{1-\gamma} & \text{if } \gamma > 1; \\ \log x & \text{if } \gamma = 1. \end{cases}$$

(To take into account the inelastic intertemporal substitution, we exclude the case $0 \leq \gamma < 1$.)

Then the optimal consumption is given

$$c_{L,t+L} = \frac{\sum_{i=L}^{\mathbf{I}_t} \rho_{t+L,t+i}^{-1}\ y_{i,t+i}}{\sum_{i=L}^{\mathbf{I}_t} \delta^{(i-L)/\gamma}\ \rho_{t+L,t+i}^{1/\gamma-1}\ \beta_{i,t+i}^{1/\gamma}\ m_{i,t+i}}$$

and

$$c_{i,t+i} = \delta^{(i-L)/\gamma}(\rho_{t+L,t+i}\ \beta_{i,t+i})^{1/\gamma} c_{L,t+L}, \qquad i = L+1, L+2, \ldots, \mathbf{I}_t.$$

For future use (e.g. at the reoptimization after shocks), it will be worthwhile to derive another form of optimal consumption path, which does not distinguish between initial and continued consumptions. On the other hand, it also relies on $a_{i-1,t-1}$ defining the capital value of the remaining life path. Because $\beta_{L,t} = 1$ but $\beta_{i,t}$ may be different from 1, $1/\beta_{i,t}$ appears in the denominator.
Consumption at age i in period t

$$c_{i,t} = \frac{R_t a_{i-1,t-1} + \sum_{j=i}^{\mathbf{I}_{t-i}} \rho_{t,t+j-i}^{-1}\ y_{j,t+j-i}}{\sum_{j=i}^{\mathbf{I}_{t-i}} \delta^{(j-i)/\gamma}\ \rho_{t,t+j-i}^{1/\gamma-1}\ (\beta_{j,t+j-i}/\beta_{i,t})^{1/\gamma}\ m_{j,t+j-i}}$$

and determine $a_{i,t}$ and $c_{i,t+1}$, respectively.

A complex life-cycle model

Having discussed the simple life cycle block, it is time to introduce the complexities: habit formation, inheritance and credit constraint.

Since the bulk of the income is wage, and the other components like pensions and bequests more or less also follow wage dynamics, we assume the following *habit formation* mechanism. Optimizing their consumption paths, people care for relative rather than absolute consumption values. Therefore the utility function should also reflect the secular increase in productivity (see Carroll (2000) for habit formation). The simplest modeling of this phenomenon is to generalize by a utility function, where the per capita consumption $c_{i,t+i}$ is discounted by the productivity level g^{t+i}

$$u_{i,t+i}(c_{i,t+i}) = \beta_{i,t+i}\ m_{i,t+i}\ u(c_{i,t+i}/g^{t+i}).$$

Hence

$$c_{i,t+i} = \delta^{(i-L)/\gamma}(\rho_{t+L,t+i}\ \beta_{i,t+i})^{1/\gamma} c_{L,t} g^{i-L}, \qquad i = L+1, L+2, \ldots, \mathbf{I}_t,$$

where the initial adult consumption is given by

$$c_{L,t+L} = \frac{\sum_{i=L}^{\mathbf{I}_t} \rho_{t+L,t+i}^{-1}\ y_{i,t+i}}{\sum_{i=0}^{\mathbf{I}_t} \delta^{(i-L)/\gamma}\ \rho_{t+L,t+i}^{1/\gamma-1} \beta_{i,t+i}^{1/\gamma}\ m_{i,t+i}\ g^{i-L}}.$$

Taking into account that $\beta_{i,t}$ may differ from 1, the recursive form of consumption at age i in period t is modified:

$$c_{i,t} = \frac{R_t a_{i-1,t-1} + \sum_{j=i}^{\mathbf{I}_{t-i}} \rho_{t,t+j-i}^{-1} y_{j,t+j-i}}{\sum_{j=i}^{\mathbf{I}_{t-i}} \delta^{(j-i)/\gamma} \rho_{t,t+j-i}^{1/\gamma-1} (\beta_{j,t+j-i}/\beta_{i,t})^{1/\gamma}\ m_{j,t+j-i}\ g^{j-i}}$$

and calculate $a_{i,t}$ and $c_{i,t}$ alternatingly.

The second complication is connected with the inheritance. As is well-known, parents leave *bequest* to their children. If we do not want to complicate superfluously our analysis with the infinite chain of bequest motives á la Barro (1974), we must find a simple solution. Denoting the adult's lifetime discounted earnings, starting to work at period t as

$$\bar{w}_{t+L} = \sum_{j=L}^{J_t} \rho_{t+L,t+j}^{-1}\ w_{j,t+j}$$

and assuming that each parent *leaves* a given share (say κ) of this variable at his death in $t+\mathbf{I}_t$ as a bequest, i.e. $\bar{q}_{t+L} = \kappa \bar{w}_{t+L}$, calculated also as a capital value in $t+L$. In current value, it is given by

$$q_{t+\mathbf{I}_t} = \bar{q}_{t+\mathbf{I}_t} \rho_{t+L,t+\mathbf{I}_t}.$$

Naturally, his father also left a similar bequest with capital value in period $t-H+L$ when the heirs were of age $F_t = \mathbf{I}_{t-H} - H$. Then $\bar{q}_{t-H+L} = \kappa \bar{w}_{t-H+L}$. Since it was divided among f_t heirs, the capital value in $t+L$ of the bequest *received* is

$$\bar{q}^*_{t+F_t} = \frac{\kappa \bar{w}_{t-H+L}}{f_t} \rho_{t-H+L,t+L}.$$

The current value of bequest *received* (in period $t+F_t$) is

$$q^*_{t+F_t} = \bar{q}^*_{t+F_t} \rho_{t+L,t+F_t}.$$

Denote by $\hat{y}_{i,t+i}$ the *broad income* in period $t+i$ which is the sum of standard income plus the signed bequest (bequest received has a positive, bequest left has a negative sign):

$$\hat{y}_{i,t+i} = y_{i,t+i} + \begin{cases} q^*_{t+F_t} & \text{if } i = F_t; \\ -q_{t+\mathbf{I}_t} & \text{if } i = \mathbf{I}_t; \\ 0 & \text{otherwise.} \end{cases}$$

Then the earlier identities are modified:

$$a_{i,t} = R_t a_{i-1,t-1} + \hat{y}_{i,t} - m_{i,t} c_{i,t}$$

and

$$s_{i,t} = a_{i-1,t-1}(R_t - 1) + \hat{y}_{i,t} - m_{i,t} c_{i,t},$$

respectively. With this definition, the previous formula remains valid, only a hat should be put on $y_{i,t}$:

$$\hat{c}_{i,t} = \frac{R_t a_{i-1,t-1} + \sum_{j=i}^{\mathbf{I}_{t-i}} \rho_{t,t+j-i}^{-1}\ \hat{y}_{j,t+j-i}}{\sum_{j=i}^{\mathbf{I}_t} \delta^{(j-i)/\gamma}\ \rho_{t,t+j-i}^{1/\gamma-1} (\beta_{j,t+j-i}/\beta_{i,t})^{1/\gamma}\ m_{j,t+j-i}\ g^{j-i}}.$$

A third complication is the presence of *credit constraint*: the assets cannot be negative (Hubbard and Judd, 1986 and Hubbard et al., 1995): $a_i \geq 0$, $i = 0, 1, 2, \ldots, \mathbf{I}_t - 1, \mathbf{I}_t$. Credit constraints are especially stringent when children need to be feed from low starting earnings, further diminished by significant public pension contributions. Note, however, that Hubbard et al. neglected the "family composition changes" (p. 393).

The optimization problem under credit constraint is not that simple but in our numerical setting, we have found a quite simple and satisfactory algorithm.

Take the earlier of the date of the arrival of the bequest or the period when the children start to work: $K_t = \min(t + F_t, t + L + H)$. Let $V_t = \min(F_t - 1, L + H - 1)$. Cut the optimization process into two by K_t.

(1) Calculate the model for the periods $[t + L, K_t - 1]$ or ages $[L, V_t - 1]$

(2) Calculate the model for the periods $[K_t, t + \mathbf{I}_t]$ or ages $[V_t, \mathbf{I}_t]$ with bequest received at the end K_t and bequest left at the end $t + \mathbf{I}_t$.

We can unify the two cases by introducing the notation

$$M_{i,t} = \begin{cases} V_{t-i} & \text{if } L \le i \le V_{t-i}; \\ \mathbf{I}_{t-i} & \text{if } V_{t-i} < i \le \mathbf{I}_{t-i}. \end{cases}$$

Here the broad incomes $\hat{y}_{i,t+i}$s are helpful, since the bequest to be received and left are incorporated into incomes. The generalized formula runs as follows:

$$\hat{c}_{i,t} = \frac{R_t a_{i-1,t-1} + \sum_{j=i}^{M_{i,t}} \rho_{t,t+j-i}^{-1}\ \hat{y}_{j,t+j-i}}{\sum_{j=i}^{M_{i,t}} \delta^{(j-i)/\gamma} \rho_{t,t+j-i}^{1/\gamma-1}\ (\beta_{j,t+j-i}/\beta_{i,t})^{1/\gamma}\ m_{j,t+j-i}\ g^{j-i}}$$

and determine $a_{i,t}$ and $M_{i,t}$, respectively.

Note that in our heuristic solution, the assets may become slightly negative around V_t. The simplest way is to say that such small credits are possible. The perfect solution would demand a much more complex algorithm but we skip it.

Initial asset conditions are given as $(a_{i,-1})$. Of course, the simplest case is to assume that they were the result of previous optimization, most notably in a steady-state one. In more detail, we must go back to $-I_0$, Consider the optimization procedure of the consumer who was born in $t = -I_0$ and entered the labor market in $t = L - I_0$ with asset $a_{L-1,L-I_0-1} = 0$, Solving the optimization problem yields the asset path $(a_{i,-I_0+i})_{i=L}^{I_0}$, which can be converted into asset profile

$$(a_{i,-I_0})_{i=L}^{I_0} = (a_{i,-I_0+i}/g^{i-L})_{i=L}^{I_0}.$$

Shocks

Until now we have neglected the shocks which may hit the system and necessitates subsequent reoptimization. Now we fill up the gap. Suppose that the government abruptly changes its policy parameters τ_t, ι_t and θ_t. Denote the changed values by tilde. For notational simplification, in $\tilde{y}_{i,t+i}$ will drop the hat. Then the workers and pensioners must also change their remaining consumption paths. Due to the changes in the policy paths, the income $y_{i,t+i}$ also changes for $t \ge T$.

The shocked optimum at T is as follows:

$$\tilde{c}_{i,T} = \frac{R_T a_{i-1,T-1} + \sum_{j=i}^{M_{i,T}} \rho_{T,T+j-i}^{-1}\ \tilde{y}_{j,T+j-i}}{\sum_{j=i}^{M_{i,T}} \delta^{(j-i)/\gamma}\ \rho_{T,T+j-i}^{1/\gamma-1}(\beta_{j,T+j-i}/\beta_{i,T})^{1/\gamma}\ m_{j,T+j-i}\ g^{j-i}}$$

and determine $a_{i,T}$s.

There are at least two ways of defining the *expected* incomes $\tilde{y}_{i,t}$: a) under *rational expectations*, the workers correctly foresee the relevant future values of θ_t, ι_t and τ_t, respectively:

$$\theta_t^r = \theta_t, \qquad \iota_t^r = \iota_t \quad \text{and} \quad \tau_t^r = \tau_t; \qquad t = 0, 1, \dots;$$

under *naive expectations*, the workers naively identify the relevant future values of θ_t, ι_t and τ_t by their trend values, respectively:

$$\theta_p^n = \theta_t, \qquad \iota_t^p = \iota_t \quad \text{and} \quad \tau_p^n = \tau_t; \qquad p = t+1, \dots, \quad t = 0, 1, \dots, .$$

For $e = r, n$, $\tilde{y}_{i,t}^e$ contains τ_v^e and θ_v^e, with $v < t$, respectively. In this version we shall use only rational expectations, but in later versions we shall also consider naive expectations.

4. Numerical results

We have formulated our model but it is so complex that we can only analyze it with the aid of a computer. To simplify the presentation, as a prelude, in this version, we use decades rather than years.

Basic run

First we describe our basic run. We start displaying our numerical results with the *demographic block*. Assume that the end of childhood and the age at birth are as follows: $L = 2$, $H = 3$.

Let start the dynamics in $t = 0$ (calendar time 1950) and assume that the previous 7 decades have time indices $t = -7, -6, \dots, -1$.

Fertility started to diminish uniformly in $t = 2$ (1970) from 1 to 0.79 in 3 decades, i.e. ended in $t = 5$ (2000). In equation:

$$f_t = \begin{cases} f1, & \text{if } t < T1^f; \\ f1 + \Delta f(t - T1^f) & \text{if } T1^f \le t \le T2^f; \\ f2, & t > T2^f, \end{cases}$$

where $f2 - f1 = \Delta f(T2^f - T1^f)$.

Numerical values: $f1 = 1 > f2 = 0.79$, $\Delta f = 0.1$, $T1^f = 2$, $T2^f = 5$.

Life expectancy jumped in year T^I from 6 $I1$ to $I2$.

In equation:

$$I_t = \begin{cases} I1, & \text{if } t < T^I; \\ I2, & \text{if } t \ge T^I. \end{cases}$$

The parameter values are as follows: $I1 = 6$ and $I2 = 7$, $T^I = 5$. For better understanding, we spell out this change: I_t jumps from $I_4 = 6$ in 1990 to $I_5 = 7$ in 2000.

For the time being, we set the retirement age low and time invariant: $J_{0,t} = 5$ but later on we shall modify it.

We shall assume that the initial population was stationary:

$$n_{0,-I_0} = n_{0,-I_0+1} = \ldots, n_{0,-1} = 1$$

and the fertility was unitary: $f_{-I_0} = \cdots = f_{-1} = 1$.

The left part of Table 1a displays the size of the subpopulations of children, workers and pensioners. In our setup, the jump in life expectancy only delays but does not counterbalance the drop in fertility, and the total population begins to decrease again. The real problem is the spectacular rise in the share of pensioners.

Making use of the fact that the pension block can be solved without solving the consumption block, first we concentrate on the former.

Let us assume that the wage-dynamics can be described by a geometrical series with a quotient ε:

$$w_{i,t} = \varepsilon^{i-L} g^{t-L}, \qquad i = L+1, \ldots, J_t.$$

We shall work with a modest $\varepsilon = 1.005^{10}$. Having chosen $\alpha = 1.015^{10}$, and $g = 1.0175^{10}$, we can now determine the aggregate total wage and the interest factor series.

Now we determine an equilibrium contribution rate τ_t for $t = -7, -6, \ldots, 0, 1$. Because of aging, from $t = 2$ (1970), the equilibrium contribution rate τ_t^{o} will rises from 0.13 to 0.292 by $t = 7$ (2020) and then oscillates a little bit around 0.27. Finally, let us underline the steep rise of the implicit pension debt in terms of total wages during 1930 and 2010: from 0.213 to 0.686. It needs further discussion why this ratio is stabilized at a lower level 0.571. (Note that in our decade model, this stock/flow ratio is much lower than it would be in an annual model.)

Turning to the consumption path (Table 1b), we must define the parameters of the utility function: $\gamma = 2 =$ elasticity of intertemporal substitution, $\beta = 0.7 =$ utility correction, $\delta = 1/R_A = 1/(\alpha g) = 0.9682768 =$ discount factor. In addition, $\mu = 0.6 =$ equivalent consumption coefficient. Rising contribution rates reduce individual net incomes. This reduction implies proportionally diminished consumption and saving decisions. For example, in $t = 0$, the adult consumption profile corresponds to the steady-state optimum. (Numbers are given in terms of the initial total wages of the youngest worker in $t = 2$, our numeraire). In the next decades, the profile adjust itself to the new circumstances. The consumption profile stabilizes around $t = 13$ (2080). Asset dynamics (Table 1.c) is a simple consequence of the income and consumption paths. This becomes really important when privatization and prefunding will be studied.

Alternative policies

Consider other scenarios, where the government realizes in $t = 5$ (2000) that a demographic change has been taking place since $t = 2$. Alternative pension reforms are introduced in $t = 6$ (2010). We shall study three different policies: 1. decreasing the accrual rates, 2. replacing wage indexation of pensions with price indexation and 3. raising the retirement age. For the time being it will be assumed that all the policy changes are correctly anticipated by the public but in future versions we will study unanticipated changes as well.

1. Decreasing the accrual rates

To avoid duplication, from Table 2.a, the numbers of the unchanged demographic block are dropped. Here the pension accrual is immediately decreased from $\theta_t = 0.015\times$

10 to $\theta_T = 0.0125 \times 10$ in $t = 6$, while keeping retirement age constant. Note that with the drop in per capita pension, the contribution rate stabilizes at a lower value, namely at 0.241. Similarly, the relative implicit pension debt also diminishes from 2020, and faster than in the base run. Tables 2.a–2.b contain the details.

For example, the initial pension benefits are lower than in the base run after 2010: $b_{60,2020} = 1.170 < 1.220$, moreover, $b_{60,2100} = 3.933 < 4.495$. Similarly, the final contribution rate drops from 0.27 to 0.24. Consumption values differ much less: at the beginning, initial consumption drops with respect to the base run: $c_{20,2010} = 1.221 < 1.1439$; but at the end, it more than recovers: $c_{20,2100} = 6.181 > 5.902$.

3. Replacing wage indexation by price indexation

Here the wage indexation rule is replaced by a price indexation rule in period $t = 6$ (2010). Tables 3.a–3.b contain the relevant data, reinstating the changed demographic block. Similarly to the reduced accrual rate, the moderation of indexation also reduces the pensions and the contribution rates.

4. Raising the retirement ages

Here the retirement age is immediately raised from 5 to 6 (in 2010). Tables 4.a–4.b contain the descriptions.

The radical rise in the retirement age restores and even improves the pensioner-to-worker ratio of 2/3.8 in 2000 to 1/4.58 in 2010. Small wonder that the contribution rate almost returns from the very high 0.274 to the original value, now 0.16. At the same time, longer employment increases per-capita pensions (from 4.49 to 5.17 in 2100), sustaining the high relative implicit pension debt. This already shows that the two measures (reducing promises and increasing the burden) should be combined.

The consumption also increases with respect to the base run: $c_{20,2010} = 1.46 > 1.216$ and it widens to $c_{20,2100} = 6.85 > 5.9$.

5. Conclusions

We have just started to write our programs, therefore they do not take into account important details: policy surprises and fine demographic details. We must work a lot until we arrive to more definite results. Nevertheless, our early experiments have already testified the power of our approach: we have obtained meaningful results on the qualitative differences between no policy change and radical reforms.

Appendix: Some proofs

The Appendix contains two parts: the derivations of pension wealth profiles and of the optimal consumption path.

Pension wealth profiles

We shall need cross-sectional profiles rather than longitudinal path. To do so we must deduct $J + 1$, j and i respectively from the corresponding indices starting with t.

Benefit in time t

$$b_t = g^{t-L} \sum_{j=L}^{J} \theta_{t+j-J-1}(1-\tau_{t+j-J-1}) w_{j,L}.$$

Pension wealth for workers

$$d_{j,t} = g^{t-L-j} \sum_{h=L}^{j} \theta_{t+h-j}(1-\tau_{t+h-j}) w_{h,L} \sum_{i=\mathbf{J}_t+1}^{I} g^i \rho_{t,t+i-j}^{-1}, \qquad j = L, \dots, \mathbf{J}_t.$$

Pension wealth for pensioners

$$d_{i,t} = \sum_{h=i+1}^{\mathbf{I}_t} b_{h,t+h-i} \rho_{t,t+h-i}^{-1}, \qquad i = \mathbf{J}_t + 1, \dots, \mathbf{I}_t.$$

Finally, we discuss the simplest case, namely steady state paths with stationary population, hence $R_t = R$, $\theta_t = \theta$ and $\tau_t = \tau$. Then
Benefit

$$b_t = g^{t-L}\theta(1-\tau)E_J, \quad \text{where} \quad E_J = \sum_{h=L}^{J} w_{h,L}.$$

Using the notation E_j for d_j and inserting b_ts into d_i, yields
Pension wealth for workers

$$d_{j,t} = g^{t-L}\theta(1-\tau)E_j \sum_{i=J+1}^{I} g^{i-j} R^{h-i}, \qquad j = L, \dots, J.$$

Pension wealth for pensioners

$$d_{i,t} = g^{t-L}\theta(1-\tau)E_J \sum_{h=i+1}^{I} g^{h-i} R^{i-h}, \qquad i = J+1, \dots, I.$$

Introducing the notation $\mathbf{g} = g/R$, these formulas respectively simplify to
Pension wealth for workers

$$d_{j,t} = g^{t-L}\theta(1-\tau)E_j \mathbf{g}^{J+1-j} \frac{\mathbf{g}^{I-J} - 1}{\mathbf{g} - 1}, \qquad j = L, \dots, J.$$

Pension wealth for pensioners

$$d_{i,t} = g^{t-L}\theta(1-\tau)E_J \mathbf{g} \frac{\mathbf{g}^{I-i} - 1}{\mathbf{g} - 1}, \qquad i = J+1, \dots, I.$$

Optimal consumption paths

Here is the derivation of the optimal consumption path. Introduce the Lagrange function with a multiplier λ:

$$\mathcal{L} = \sum_{i=L}^{\mathbf{I}_t} [\delta^{i-L} u_i(c_{i,t+i}) - \lambda \rho_{t+L,t+i}^{-1} (y_{i,t+i} - c_{i,t+i})].$$

The optimal household consumption path is determined from the implicit Euler equations:

$$\delta^{i-L} \beta_{i,t+i} u'(c_{i,t+i}) = \lambda \rho_{t+L,t+i}^{-1}, \qquad i = L, L+1, \ldots, \mathbf{I}_t.$$

Comparing the multipliers for i and L, results in

$$\delta^{i-L} \beta_{i,t+i} \rho_{t+L,t+i} u'(c_{i,t+i}) = \beta_{L,t+L} u'(c_{L,t+L}).$$

Since $\beta_{L,t+L} = 1$, we drop it, but later on we shall need it again.

Substituting $c_{i,t+i}$s into the budget constrain, $c_{L,t+L}$ is determined, hence the entire consumption path is determined.

Tables

Table 1.a. *Population and pensions: Base run*

Decades t	Kids K_t	Workers M_t	Pension-ers P_t	Interest rate R_t	Accrual rate θ_t	Per cap. benefits b_t	Contribution rate τ_t	IPD/WW D_t/W_t
1930	2.000	4.000	1.000	1.033	0.150	0.281	0.130	0.213
1940	2.000	4.000	1.000	1.033	0.150	0.335	0.130	0.239
1950	2.000	4.000	1.000	1.033	0.150	0.398	0.130	0.255
1960	2.000	4.000	1.000	1.033	0.150	0.473	0.130	0.293
1970	1.930	4.000	1.000	1.033	0.150	0.563	0.130	0.364
1980	1.790	4.000	1.000	1.033	0.150	0.670	0.130	0.482
1990	1.650	3.930	1.000	1.031	0.150	0.797	0.133	0.509
2000	1.525	3.790	2.000	1.029	0.150	0.947	0.274	0.662
2010	1.414	3.580	2.000	1.027	0.150	1.076	0.283	0.686
2020	1.303	3.315	2.000	1.025	0.150	1.220	0.292	0.682
2030	1.205	3.064	1.930	1.025	0.150	1.378	0.290	0.676
2040	1.117	2.828	1.790	1.025	0.150	1.559	0.277	0.668
2050	1.030	2.619	1.650	1.025	0.150	1.853	0.269	0.670
2060	0.952	2.421	1.525	1.025	0.150	2.217	0.269	0.635
2070	0.883	2.234	1.414	1.025	0.150	2.658	0.273	0.604
2080	0.814	2.069	1.303	1.025	0.150	3.179	0.273	0.584
2090	0.752	1.912	1.205	1.025	0.150	3.786	0.274	0.571
2100	0.697	1.765	1.117	1.025	0.150	4.495	0.275	0.571

Table 1.b. *Consumption: Base run*

Decades t	cons(2) c_2	cons(3) c_3	cons(4) c_4	cons(5) c_5	cons(6) c_6	cons(7) c_7
1930	0.348	0.348	0.492	0.364	0.305	0
1940	0.414	0.414	0.586	0.586	0.363	0
1950	0.492	0.492	0.694	0.697	0.583	0
1960	0.595	0.586	0.815	0.825	0.693	0
1970	0.719	0.707	0.778	0.970	0.821	0
1980	0.870	0.855	0.926	0.926	0.965	0
1990	0.955	1.026	1.033	1.093	0.914	0
2000	1.030	1.117	1.111	1.208	1.069	1.068
2010	1.216	1.192	1.277	1.285	1.169	1.237
2020	1.439	1.391	1.489	1.462	1.230	1.338
2030	1.730	1.646	1.784	1.703	1.399	1.407
2040	2.088	1.977	2.145	2.038	1.628	1.598
2050	2.498	2.390	2.570	2.456	1.952	1.863
2060	2.964	2.856	3.058	2.939	2.349	2.232
2070	3.515	3.388	3.629	3.494	2.810	2.685
2080	4.178	4.023	4.314	4.154	3.346	3.216
2090	4.963	4.778	5.124	4.933	3.974	3.827
2100	5.902	5.672	6.090	5.855	4.717	4.542

Table 1.c. *Assets: Base run*

Decades t	asset(2) a_2	asset(3) a_3	asset(4) a_4	asset(5) a_5	asset(6) a_6	Total assets A_t
1930	0.086	0	0.087	0.020	0	0.193
1940	0.103	0	0.103	0.134	0	0.340
1950	0.122	0	0.118	0.159	0	0.400
1960	0.136	0	0.126	0.187	0	0.449
1970	0.151	0	0.483	0.214	0	0.848
1980	0.164	0	0.584	0.942	0	1.690
1990	0.272	0	0.824	1.125	1.162	3.364
2000	0.191	0	0.814	1.309	1.378	3.666
2010	0.219	0	0.973	1.444	1.617	4.139
2020	0.246	0	1.178	1.740	1.835	4.648
2030	0.279	0	1.386	2.135	2.199	5.166
2040	0.347	0	1.666	2.555	2.651	5.739
2050	0.432	0	1.991	3.076	3.168	6.366
2060	0.517	0	2.359	3.645	3.793	7.013
2070	0.609	0	2.806	4.301	4.493	7.665
2080	0.723	0	3.334	5.124	5.330	8.422
2090	0.859	0	3.962	6.083	6.349	9.268
2100	1.011	0	4.718	7.222	7.529	10.159

Table 2.a. *Pensions: Reduced accrual rate*

Decades rate t	Accrual rate θ_t	Per cap. benefits b_t	Contribution rate τ_t	IPD/WW D_t/W_t
1930	0.150	0.281	0.130	0.213
1940	0.150	0.335	0.130	0.239
1950	0.150	0.398	0.130	0.255
1960	0.150	0.473	0.130	0.293
1970	0.150	0.563	0.130	0.364
1980	0.150	0.670	0.130	0.482
1990	0.150	0.797	0.133	0.509
2000	0.150	0.947	0.274	0.662
2010	0.125	1.076	0.283	0.686
2020	0.125	1.170	0.286	0.677
2030	0.125	1.266	0.273	0.666
2040	0.125	1.371	0.250	0.655
2050	0.125	1.573	0.232	0.653
2060	0.125	1.905	0.230	0.619
2070	0.125	2.308	0.235	0.560
2080	0.125	2.778	0.238	0.519
2090	0.125	3.315	0.240	0.494
2100	0.125	3.933	0.241	0.490

Table 2.b. *Consumption: reduced accrual rate*

Decades t	cons(2) c_2	cons(3) c_3	cons(4) c_4	cons(5) c_5	cons(6) c_6	cons(7) c_7
1930	0.348	0.348	0.492	0.364	0.305	0
1940	0.414	0.414	0.586	0.586	0.363	0
1950	0.492	0.492	0.694	0.697	0.583	0
1960	0.595	0.586	0.815	0.825	0.693	0
1970	0.719	0.707	0.778	0.970	0.821	0
1980	0.870	0.855	0.926	0.926	0.965	0
1990	0.955	1.026	1.033	1.093	0.914	0
2000	1.030	1.117	1.095	1.208	1.069	1.068
2010	1.221	1.192	1.244	1.267	1.169	1.237
2020	1.463	1.397	1.444	1.424	1.213	1.338
2030	1.785	1.673	1.730	1.652	1.362	1.387
2040	2.179	2.039	2.108	1.977	1.579	1.557
2050	2.627	2.494	2.547	2.413	1.894	1.807
2060	3.120	3.004	3.037	2.913	2.309	2.165
2070	3.690	3.565	3.599	3.470	2.785	2.638
2080	4.378	4.223	4.272	4.119	3.323	3.187
2090	5.198	5.006	5.070	4.885	3.941	3.800
2100	6.181	5.940	6.024	5.793	4.670	4.504

Table 3.a. *Population and pensions: from wage to price indexation*

Decades t	Index ι_t	Per cap. benefits b_t	Contribution rate τ_t	IPD/WW D_t/W_t
1930	1	0.281	0.130	0.213
1940	1	0.335	0.130	0.239
1950	1	0.398	0.130	0.255
1960	1	0.473	0.130	0.293
1970	1	0.563	0.130	0.364
1980	1	0.670	0.130	0.482
1990	1	0.797	0.133	0.509
2000	1	0.947	0.274	0.642
2010	0	1.076	0.260	0.666
2020	0	1.229	0.269	0.664
2030	0	1.400	0.269	0.659
2040	0	1.596	0.259	0.653
2050	0	1.909	0.253	0.655
2060	0	2.277	0.254	0.620
2070	0	2.722	0.256	0.594
2080	0	3.250	0.256	0.578
2090	0	3.869	0.257	0.568
2100	0	4.596	0.258	0.569

Table 3.b. *Consumption: From wage to price indexation*

Decades t	cons(2) c_2	cons(3) c_3	cons(4) c_4	cons(5) c_5	cons(6) c_6	cons(7) c_7
1930	0.348	0.348	0.492	0.364	0.305	0
1940	0.414	0.414	0.586	0.586	0.363	0
1950	0.492	0.492	0.694	0.697	0.583	0
1960	0.595	0.586	0.815	0.825	0.693	0
1970	0.719	0.707	0.778	0.970	0.821	0
1980	0.870	0.855	0.905	0.926	0.965	0
1990	0.955	1.026	1.008	1.068	0.914	0
2000	1.046	1.117	1.095	1.178	1.045	1.068
2010	1.255	1.211	1.277	1.267	1.140	1.209
2020	1.484	1.436	1.493	1.461	1.213	1.305
2030	1.778	1.697	1.788	1.707	1.398	1.387
2040	2.137	2.032	2.142	2.043	1.632	1.598
2050	2.551	2.446	2.560	2.452	1.956	1.868
2060	3.028	2.917	3.044	2.927	2.346	2.237
2070	3.594	3.460	3.615	3.479	2.799	2.681
2080	4.275	4.114	4.300	4.138	3.331	3.203
2090	5.081	4.889	5.110	4.918	3.959	3.810
2100	6.041	5.806	6.074	5.839	4.702	4.524

Table 4.a. *Population and pensions: Retirement age jumps*

Decades t	Kids K_t	Workers M_t	Pension-ers P_t	Interest rate R_t	Accrual rate θ_t	Per cap. benefits b_t	Contribution rate τ_t	IPD/WW D_t/W_t
1930	2.000	4.000	1.000	1.033	0.150	0.281	0.130	0.213
1940	2.000	4.000	1.000	1.033	0.150	0.335	0.130	0.239
1950	2.000	4.000	1.000	1.033	0.150	0.398	0.130	0.255
1960	2.000	4.000	1.000	1.033	0.150	0.473	0.130	0.289
1970	1.930	4.000	1.000	1.033	0.150	0.563	0.130	0.346
1980	1.790	4.000	1.000	1.033	0.150	0.670	0.130	0.435
1990	1.650	3.930	1.000	1.031	0.150	0.797	0.133	0.412
2000	1.525	3.790	2.000	1.029	0.150	0.947	0.274	0.520
2010	1.414	4.580	1.000	1.056	0.150	1.392	0.136	0.540
2020	1.303	4.315	1.000	1.027	0.150	1.657	0.144	0.541
2030	1.205	3.994	1.000	1.025	0.150	1.967	0.156	0.541
2040	1.117	3.688	0.930	1.025	0.150	2.329	0.156	0.539
2050	1.030	3.409	0.860	1.025	0.150	2.758	0.156	0.543
2060	0.952	3.155	0.790	1.025	0.150	3.364	0.158	0.516
2070	0.883	2.914	0.735	1.025	0.150	3.982	0.159	0.519
2080	0.814	2.693	0.679	1.025	0.150	4.722	0.158	0.525
2090	0.752	2.493	0.624	1.025	0.150	5.613	0.157	0.528
2100	0.697	2.302	0.580	1.025	0.150	6.675	0.158	0.531

Table 4.b. *Consumption: Retirement age jumps*

Decades t	cons(2) c_2	cons(3) c_3	cons(4) c_4	cons(5) c_5	cons(6) c_6	cons(7) c_7
1930	0.348	0.348	0.492	0.364	0.305	0
1940	0.414	0.414	0.586	0.586	0.363	0
1950	0.492	0.492	0.694	0.697	0.583	0
1960	0.595	0.586	0.815	0.825	0.693	0
1970	0.719	0.707	0.823	0.970	0.821	0
1980	0.870	0.855	1.151	0.979	0.965	0
1990	0.955	1.026	1.143	1.358	0.967	0
2000	1.066	1.117	1.208	1.337	1.328	1.130
2010	1.461	1.414	1.323	1.603	1.483	1.761
2020	1.725	1.689	1.546	1.528	1.549	1.714
2030	2.039	1.975	1.841	1.770	1.463	1.773
2040	2.425	2.331	2.188	2.105	1.693	1.673
2050	2.880	2.773	2.596	2.502	2.014	1.935
2060	3.421	3.296	3.085	2.971	2.395	2.304
2070	4.069	3.911	3.670	3.526	2.842	2.738
2080	4.843	4.652	4.365	4.196	3.373	3.249
2090	5.764	5.542	5.195	4.996	4.018	3.861
2100	6.851	6.588	6.177	5.938	4.777	4.592

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