A DETAIL-FREE MEDIATOR AND THE 3 PLAYER CASE

PÉTER VIDA

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Author: Péter VIDA, Universitat Autònoma de Barcelona, IDEA
E-mail: pvida@idea.uab.es

VIDA PÉTER

EGY UNIVERZÁLIS MEDIÁTOR, ÉS A HÁROM JÁTÉKOS ESETE

Összefoglaló

Két játékos felkérhet egy független, kívülálló harmadik személyt, hogy az mediálja tárgyalásukat és így részben segítse feloldani a konfliktust. Előnyös, ha a mediátor ismeri a játékosok helyzetét és aszerint ad elfogadható javaslatokat; habár egy kívülálló informálása igen költséges lehet. Hovatovább, egy korrupt mediátor igen nagy befolyással lehet a játék kimenetelére.

Ebben a cikkben bemutatunk egy egyszerű, ellenőrizhető mediátort, aki bármelyik játékban biztonságosan alkalmazható. Pontosabban, nem szükséges, hogy a mediátor ismerje a játékot és az elérni kívánt kimenetet. Ilyen értelemben független a szituáció részleteitől, azaz univerzális. Továbbá, a játékosok könnyedén ellenőrizhetik hogy követi-e a mediálás szabályait.

Technikailag: tetszőleges teljes információs két szereplős játék, tetszőleges korrelált egyensúlya előáll a játék egy fix kiterjesztésének Nash egyensúlyaként, ahol – mielőtt akciót választanak az eredeti játékban – a játékosok kommunikálhatnak a mediátoron keresztül. A kulcs, hogy a mediált kommunikáció után résztvehetnek egy sima beszélgetésben.

A mediátor transzparens és univerzális, azaz függetlenül a játéktól és a választott korrelált egyensúlytól.

Az eredményt kiterjesztjük olyan esetekre is, ahol a harmadik személy függetlensége nem nyilvánvaló. Megmutatjuk hogyan implementálható tetszőleges három szereplős játék tetszőleges korrelált egyensúlya oly módon, hogy a játékosok egyike játssza a mediátor szerepét egy sima három szereplős beszélgetésben, az eredeti játékot megelőzően. A megoldás kulcsa, hogy a játékosok engedélyezhetik egymásnak, hogy titokban kihallgathassanak privát üzeneteket.

A fentiek robosztusak a játék információs feltételeire, azaz ugyanúgy vonatkoznak nem-teljes információs játékokra és azok kommunikációs egyensúlyaira.

Kulcsszavak: beszélgetés, kommunikációs eszköz, korrelált egyensúly, kommunikációs egyensúly, univerzális mechanizmus, mediátor

A Detail-free Mediator and the 3 Player Case

Péter Vida,*
Departament d'Economia i d'Història Econòmica
Universitat Autònoma de Barcelona, IDEA

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Abstract

Two players can make use of a trusted third party who mediates and partially resolves their conflict. Usually, the mediator should be aware of the situation and give suggestions to the players accordingly. However, a corrupt mediator can have a big influence on the outcome of the game. We single out a transparent mediator which can be safely applied in any two player game without loss of efficiency. That is, the mediator is independent of the game and the desired outcome.

Technically, we show that any correlated equilibrium of any two player game can be obtained as Nash equilibria of the game, extended with cheap, pre-play communication, where players can communicate through the proposed mediator. The key idea is that after the mediated communication the players can have a plain conversation.

In particular, the mediating communication device is transparent, controllable and is the same for all games and for all equilibrium distributions.

We extend the result to three player games and show that one of the players can play the role of the mediator. We implement the set of correlated equilibrium in Nash equilibria of an extended game where the players have a plain conversation. The central assumption is that players can be invited to eavesdrop a private conversation.

We extend the analysis to games with incomplete information and to the set of communication equilibria.

Keywords: cheap talk, communication device, correlated equilibrium, communication equilibrium, detail-free mechanism, mediator

1   Introduction

In some games, players would like to achieve outcomes which Pareto dominate the Nash equilibrium payoffs. Since Crawford an Sobel (1982) it has benn well known that, direct, cheap, preplay communication in games with incomplete information may extend the set of Bayesian Nash equilibrium payoffs. Players can achieve even better outcomes if their communication is mediated[1]. Moreover, there are situations when direct cheap talk cannot but mediated talk does help to improve upon the equilibrium outcomes (Myerson 1991, Mitusch and Strausz 2000). Clearly and intuitively, the "behavior" or the mechanism of the mediator has to vary with the situation he faces if he wants to drive the players to the best achievable outcome.

A mediator can be thought of as a third party or an institution. We are interested in the existence of a controllable and transparent mediator who can assist the players in many general games, yet acts independently of the situation.

For expositional purposes, we first we show our question and our answer for games with complete information, then we turn to the case of incomplete information.

1.1   Complete information

In games with complete information with the help of a mediator the players can achieve payoffs which give them higher welfare than any Nash equilibrium. Think of a provision of a discrete public good with externality. In case of two contributors the situation can be modelled as: the game Γ of chicken (Cavaliere 2001). Players would like to play the action

Player 2

		0	1
Player 1	0	6,6	2,7
	1	7,2	0,0

Figure 1: The chicken game

profile (0,0), which gives them the highest welfare, though it is not a Nash equilibrium of the game[2]. This can be partially achieved if one extends Γ to a two stage game Γ^μ where in the first stage a mediator randomizes over the action profiles according to some distribution μ and then sends private messages to the players. These private messages can be interpreted as suggested actions. In the second stage players choose an action in Γ. If the players follow the suggestion then μ is called a correlated equilibrium distribution (CE)(Aumann 1974) of Γ. The set of correlated equilibrium payoffs is the largest set of

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[1]The equilibrium does not constrain the informed player to be indifferent of sending different messages

[2]In the mixed strategy equilibrium (0,0) has a probability 4/9 but the profile (1,1) is also played with probability 1/9. See Ashlagi, Monderer and Tenneholtz (2005) on the value of correlation.

	0	1
0	1/2	1/4
1	1/4	0

Figure 2: The distribution μ

non-cooperative solutions when an arbitrary means of communication is available. The distribution in Figure 2. gives the highest possible utilitarian welfare to the players among the distributions which satisfy these incentive compatibility conditions -that is, among the self-enforcing agreements which allow the use of a mediator. By the help of such a mediator the players can coordinate their actions. Moreover, such an extension of a game enlarges the set of equilibrium payoffs. Our problem is that Aumann's mediator:

1. has to be tailor-made to the game and to the desired distribution at hand

2. is not transparent that is, the players may not realize, that the mediator deviated from the prescribed randomization

3. obtains knowledge about the players' action that is the players have to give up their privacy

In the paper we concentrate on the first problem and in Theorem 1 we show the existence of a unique mediator which applies for all finite 2 player games and for all distribution.

This is important, because otherwise the mediator should know the payoffs of the game and the distribution the players would like to implement. In many situations it is costly to inform a third party about the circumstances, the possible outcomes and about the preference relations over the outcomes -basically about the game. It is also unclear what are the principles which drive the mediator's choice among the possible CE distributions, even if he knows the game. Another drawback is that the players have to trust the mediator that he is randomizing according to some pre-agreed distribution.

A natural question is whether there exists a *common, transparent*, deterministic mediator who can help the players to implement any CE of any game.

In doing so, the players do not have to inform the mediator about the game. Moreover, the mediator does not have the power to impose an arbitrary distribution on the players. Players have the freedom to generate any distribution they like to.

This is the idea of Wilson's (1987) "*detail-free*" mechanism[3]. In Theorem 1 we show the existence and the functioning of a detail-free mediator.

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[3]Thanks to Andrew Postlewaite for pointing out this connection with the literature.

Moreover, in Remark 1 it turns out trivially that our mediator is transparent and controllable. More precisely, players can test whether the mediator behaves according to the rules of the mediation or not. We also hint that the players can communicate through the mediator in a way that no information leaks to the mediator about the outcome of the correlation.

These observations lead us to Theorem 2, where the mediator is no longer considered to be an independent third party, but he can have preferences over the outcomes of the game. Consider the following situation, where a mediator prefers if the public good is not provided at all and its expectation from the correlated equilibrium μ is 0. Can player 1

$$\begin{array}{cc} & \text{Player 2} \\ & \begin{array}{cc} 0 & \quad\quad 1 \end{array} \end{array}$$

$$\text{Player 1} \begin{array}{c} 0 \\ 1 \end{array} \begin{array}{|c|c|} \hline 6,6,1 & 2,7,-1 \\ \hline 7,2,-1 & 0,0,\ 2 \\ \hline \end{array}$$

Figure 3: Third coordinates may express the preferences of the mediator

and player 2 apply a mediator with such preferences? Our answer is positive!

What if the mediator is also allowed to take action? In Theorem 2 we show how to generate all the correlated equilibria of 3 player games in a fashion that one of the players can play the role of the mediator.

For Theorem 1, we take the extreme case when the mediator cannot randomize and cannot send private messages to the players. In this way, in some sense we also minimize the duties and the abilities of the mediator. All we assume is that the mediator can receive private messages and announce public ones according to a fixed deterministic function of the input messages.

We are looking for an extended game Γ^d, where before playing in Γ, players can send private messages repeatedly to a public, deterministic, minimal, detail-free mediator. A strategy in the extended game consists of two parts:

1. the communication strategy, according to which the players communicate

2. the decision rule, which chooses an action in Γ given the communication history

A Nash equilibrium of Γ^d induces a distribution on the action profiles of Γ. Consider the *AND* mediating communication device in Figure 4. which receives private inputs 0 or 1 from each of the two players, and produces a public output 1 if both inputs were 1, and 0 otherwise. We show that the *AND* mediator is detail-free in the following sense:

let Γ^d be the following extensive form game:

4

	0	1
0	0	0
1	0	1

Figure 4: A detail free mediator: the AND communication device

1. two players are allowed to communicate repeatedly through the mediator AND

2. after the *mediated* communication phase endogenously terminates, the players engage in a *direct* repeated communication where the messages are sent simultaneously

3. finally the players choose an action in Γ

Theorem 1:***Detail-freeness:*** *By using the AND communication device, any correlated equilibrium of any two player finite normal form game Γ with complete information having higher expected payoffs than the individually rational ones can be approximated as ϵ-Nash equilibria of Γ^d.*

That is, no matter what the game and the distribution is, the players can implement any correlated equilibrium with the help of the AND mediator.

Remark 1:***Transparency, secrecy***: *If the* AND *device malfunctions the players are able to realize it. Moreover the communication strategies can be chosen in a way that no relevant information leaks to the mediator. The players can mask their communication by making it seem like random noise.*

This is important because if the players notice the malfunctioning, they have the chance to restart the procedure. Moreover the mediator cannot get any knowledge about the actual play.

Lehrer (1991) studies the AND signalling structure in the context of infinitely repeated games with imperfect monitoring. He shows that any correlated distribution can be generated by a jointly controlled correlation round of "communication". Deviations in this round can be statistically detected and eventually punished. However Gossner and Vieille (2001) show that, in the case of a one-shot game extended by repeated (possibly infinite) communication through the AND no correlation can be achieved securely[4].

The main contribution of this paper in Theorem 1 is showing that direct communication after the repeated AND mediated communication phase resolves the sharp contrast between the two results above. We apply Lehrer's (1991) protocol and show that by repetition it becomes incentive compatible even in the case of one-shot games. The main idea

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[4]Secureness requires that the same strategies maintain equilibrium in any game that admits a distribution as a CE (Gossner 1998).

is that by direct communication the players can perform a jointly controlled lottery (Aumann et al. 1995) over the instances of the jointly controlled correlations. That is, players can jointly select one of the correlation rounds with uniform probabilities in a way that no unilateral deviation can affect the randomness of the lottery. If no deviation was detected in the communication phase the players play according to the selected round. Otherwise the deviator is minmax-ed.

In the three players case, Ben-Porath (2003) suggests a protocol[5] where, in case of deviation, the identity of the deviator is not revealed. However, a pair of players one of which is the deviator can be identified. Thus an effective threat has to punish both players in that pair. To maintain sequential rationality, 3 dominated Nash equilibria are needed, where each equilibrium is used to punish a pair of players. Ben-Porath (2003) shows how to implement such dominant CEs in sub-game perfect equilibria. If we set aside sequential rationality Ben-Porath's result can be sharpened with an additional assumption on the players' can communication.

To avoid situations where the deviator cannot be identified we make an additional assumption on the cheap talk procedure that the players can perform. We call a set of protocols *cheap talk with blind carbon copies (Bcc)* if in each stage players can send public and private messages to each other simultaneously, and before each stage player 1 for instance can decide to send to another player a blind carbon copy (Bcc) of the message to be sent to 3. The eavesdropping happens without the knowledge of the third player[6]. Then Theorem 1 and Remark 1 drives the following:

Theorem 2: *If the number of players is 3, the players can implement any rational[7] correlated equilibrium in Nash equilibria of the game extended by cheap talk with blind carbon copies.*

Notice that the we have Nash equilibrium and not just an ϵ one, however the protocol contains non-sequential elements and so we loose sub-game perfection. It is also important that we do not use verification of past messages nor urns and balls, just the possibility of sending private and public messages and private messages with Bcc. Eavesdroppers work as ear-witnesses of the invitor credibility. In case of deviation a deviating player cannot claim that an other player was deviating if the innocent player has a witness.

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[5]The protocol is constructed for games with incomplete information to implement communication equilibria.

[6]Think of a situation of sending an email. If a player Bcc another one and sends the mail to the third player, then the player in the Bcc receives the mail without the third player knowing it.

[7]The entries of the distribution are rational numbers.

1.2 Incomplete information

In this subsection we hint that the *AND* mediator can help two players in case of games with incomplete information as well.

In Crawford and Sobel (1982) an informed player has the possibility to partially reveal his private information by sending a message to another player who has to choose an action. The Bayesian equilibrium payoff of this game Pareto improves the equilibrium of the "silent" game where the informed player cannot communicate.

Imagine that the players can communicate through a mediator in the situation above. The informed player sends information about his type to the mediator and not to the receiver. The mediator selects an action from a distribution which depends on the senders' declaration. The mediator suggests the selected action to the receiver. In this manner the players may improve even upon the unmediated solution (see an intuitive example in Mitusch and Strausz (2000)).

In games with incomplete information the largest set of non-cooperative solutions achievable when arbitrary means of communication are available is the set of communication equilibria (Forges 1986, 1990, Myerson 1982). This is the counterpart of the set of correlated equilibria. The main difference is that here, the players can send information to the mediator who randomizes over the action profiles according to a function q of the sent messages. If q is such that the players reveal their true types and follow the suggestion of the mediator then it is a communication equilibrium. Let $D(\Gamma)$ denote the set of communication equilibrium payoffs of a game Γ with incomplete information.

Forges (1990): *For any finite game Γ with at least 3 players, every payoff in $D(\Gamma)$ is a* correlated equilibrium *payoff of the extended game, where after having received their information (as in Γ), the players can communicate*[8].

The important message is that a mediator does not have to have information about the players' type to achieve the maximal efficiency if the players can communicate after the correlation phase. It is enough if the mediator works as a correlation device.

As we have seen in Theorem 1, we can mimic any correlation device with the *AND* mediator if there are punishment payoffs for the players. In Bayesian games one cannot minmax a player after the types are manifested as private information. We take a weaker version of Ben-Porath's (2003) punishment strategies to be able to apply Theorem 1 for Bayesian games.

A communication equilibrium q is Nash dominant if there are Bayesian Nash equilibria $s(i)$ for each player i such that the conditional expected utility of each type of player i in

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[8]Assuming that players can get and send messages from an interval.

$s(i)$ is smaller than that of in q. First we have a weaker version (in terms of the implemented set) of Forges (1990) for the 2 player case:

Theorem 3:*Every Nash dominant q communication equilibrium payoff of a two player, finite Bayesian game Γ is a correlated equilibrium payoff of an extended game, where after having received their information as in Γ, the players can communicate directly.*

Now consider the following extended game with incomplete information Γ^d:

1. the players learn their types, as in Γ

2. communicate repeatedly through the *AND* communication device

3. plain communication,

4. choose an action in Γ

Then as a corollary of Theorem 1 and Theorem 3 we have:

Corollary 1: *A ϵ-Bayes-Nash equilibria of Γ^d induces q of Γ.*

This means that, by the use of the *AND* mechanism, two players can achieve outcomes which in some cases Pareto dominate the payoffs generated by plain conversation characterized in Aumann and Hart (2003) and Amitai (1996).

As a consequence of Theorem 2 and Forges (1990) we have:

Corollary 2: *If the number of players is 3, the players can implement any Nash dominant rational communication equilibrium in Bayesian equilibria of the game extended by cheap talk with Bcc.*

This means that, if players can send Bcc messages, the generated payoffs are not constrained by the level on which two players can be punished simultaneously. That is, the implementable set is bigger than that of Ben-Porath (2003).

The paper is structured as follows. Section 2 shows an easy example in the case of the chicken game and introduces some terminology for Theorem 1. In section 3 we go to the general notation and concepts and we state Theorem 1 and Theorem 2. In Section 4 we generalize the main result in games with incomplete information, Theorem 3. Section 5 contains the proof of Theorem 1 accompanied by an elaborated example due to Lehrer (1991). In the proof of Theorem 2 we use technics from the proof of Theorem 3. That is why Section 6 proves Theorem 3 followed by Section 7 proving Theorem 2. Finally we conclude and give a discussion on games extended by cheap talk procedures.

2 Example

We take a simple 2×2 game and one of its correlated equilibria. First we show how Lehrer's protocol generates the desired distribution when the players do not deviate from the prescribed randomization. Second we stress that players have the incentive to deviate and by manipulating the protocol induce a distribution that is more favorable for them. Then we show that by repeating the procedure such deviations are going to be detectable statistically and could be punished. Another problem arises by the repetition of the protocol. Namely the players cannot coordinate on which instance of the repetition they should play. Obviously, none of them can suggest one of the instances nor can they agree in advance. The problem is solved by the direct communication phase. This gives the players the possibility to jointly choose a correlation round of the mediated phase and coordinate their actions accordingly. The extended game which induces the desired distribution contains 4 functionally different parts:

1. the mediated communication phase: the repetition of the jointly controlled correlation round (Lehrer's protocol)

2. the direct communication phase:

   (a) the jointly controlled lottery over the instances of the correlation rounds

   (b) the reporting phase: where the players reveal all their past messages which were sent during the mediated phase but the ones corresponding to the correlation round chosen by the lottery

3. taking an action

Consider the chicken game[9] again as in Figure 1. and one of its correlated equilibrium distribution Figure 2.

The players send their "intended actions" privately to the mediator, who notifies them (announcing publicly 1) in case the intended action-profile was $(1, 1)$. That is by using the *AND* on Figure 4. he computes the public signal.

Notice that whenever a player sends 1, she will be able to infer the message what the other player has sent. On the other hand if she sends 0 she gets no information about the other's private message.

Step 1 : *Jointly Controlled Correlation*

Let the players send messages until the first 0 public announcement is made. We say that this *communication round* of length $p = 1$ was *successful*. Consider the communication strategies such that the players are randomizing between 0 and 1 with probability $(\frac{2}{3}, \frac{1}{3})$. The induced distribution on the message profiles is in Figure 5. Hence conditional on the

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[9]The game is due to Aumann and the example is due to Lehrer.

	0	1
0	4/9	2/9
1	2/9	1/9

Figure 5: The distribution with randomization $(\frac{2}{3}, \frac{1}{3})$.

event that the public announcement is not 1, the distribution over the sent message-profiles is exactly μ. So, the players could peg their strategies to their available information (their

	0	1
0	1/2	1/4
1	1/4	0

Figure 6: The distribution conditional on the public message being 0.

own private message and the public 0) and play the correlated equilibrium μ by simply playing as their intended actions were at the successful round. For example, if the row player sent 0 in the successful round, he believes that the column player sent 0 or 1 with probabilities $\frac{2}{3}, \frac{1}{3}$ respectively. In this case the row player wishes to play his intended action 0. On the other hand , if the column player sent 1 in the successful round, he knows for sure that the row player sent 0. In this case the column player wants to play his intended action 1.

Manipulable

Obviously, this protocol is manipulable, since for example the row player has incentives to send 1 always, and play his preferred equilibrium (1,0). Notice that doing so the induced theoretical distribution on the message-profiles is changed to:

	0	1
0	0	0
1	2/3	1/3

Figure 7: The distribution when the row player sends 1 with probability 1.

Repetition

To make the protocol incentive compatible we need to make these deviations detectable. So we suggest the following. The players repeat the above process, that is randomize again with $(\frac{2}{3}, \frac{1}{3})$ until another successful round occurs. This is going to be the second successful round. The exact time of the first and second successful rounds are random variables (κ_1, κ_2). Now let players repeat the mediated communication until say $T = 16$ successful communication round has occurred.

Statistical Detection

Now the players can perform a standard statistical test about the randomization of the opponent. This is due to sending the message 1 reveals the other's message. Put it simply, the ratio of the public announcements 1 has to be close to $\frac{1}{9}$. If T is big enough any deviation in the above randomization is statistically detectable.

Step 2a : Jointly Controlled Lottery

The players should coordinate on one of the successful rounds κ^* and play accordingly. After the mediated communication phase letting the players communicate directly allows them to jointly select one of the successful rounds. This can be done by a simple jointly controlled lottery on $\kappa_1, \ldots, \kappa_T$. By mixing their messages with probabilities $(0.5, 0.5)$ for $O = \log T = \log 16 = 4$ times repeatedly. If the resulting direct communication sequence was:

$$(0,0), (1,0), (1,1), (0,1)$$

the "interpretation" of these message pairs can be fixed as follows:

$$(0,0) = 0, (1,0) = 1, (1,1) = 0, (0,1) = 1$$

Now if we map this 4 digit (0101) to a decimal number:

$$1 + (0 * 2^0 + 1 * 2^1 + 0 * 2^2 + 1 * 2^3) = 1 + 10 = 11$$

one can easily see that the players can select randomly a number between 1 and 16. It is also clear that none of the players can influence the randomness of the selection.

Step 2b : Reporting Phase

Because of the relatively[10] simple structure of the distribution we can skip the reporting phase since all the possible deviations are detectable during the mediated phase by the repetition[11].

Step 3 : Decision Rule

We define the players decision rules after the communication terminates as: "Choose the action you intended to move" in the:

$$1 + (0 * 2^0 + 1 * 2^1 + 0 * 2^2 + 1 * 2^3) = 11th$$

successful round of the mediated communication, that is move according to the private message sent in the $\kappa^* = \kappa_{11}$th round of the whole mediated communication, if no deviation was detected. Otherwise play 1 that is, your punishment strategy.

Notice that the direct communication round in fact selects a successful communication

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[10]Relative to the structure of the communication device.

[11]The proof contains an example which shows the necessity of the reporting phase, that is there can be deviations which cannot be detected statistically during the mediated phase

round with equal probabilities in a non-manipulable way that is, selects an action profile with probability distribution close to μ.

Since the min max payoffs are always less than or equal to the correlated equilibrium payoffs no players have incentives to deviate from the prescribed communication strategy and decision rule. Moreover if no deviation took place then the players conditional beliefs about the other player action (knowing their own) coincides with that of in μ.

The method can be generalized to other CE distributions. For example if the players like to implement the distribution in Figure 8. all they have to do is to change their ran-

	0	1
0	1/3	1/3
1	1/3	0

Figure 8: The distribution with randomization (0.5,0.5).

domization in the mediated phase to $(0.5, 0.5)$. One can see that the induced conditional distribution is continuous in the communication strategies that is, player can induce distributions with irrational entries as well.

More "complicated" distributions, for example

	0	1
0	2/5	1/5
1	1/5	1/5

need more mediated stages $(p > 1)$. That is a mediated round can be longer than 1. These kind of distributions also give rise other possible deviations which cannot be detected statistically. In these cases the reporting phase can make Lehrer's protocol incentive compatible (see the details in the proof).

Let us stress again: the *AND* device can be used to generate any distribution on any finite set of action profiles.

3 Concepts and Theorem 1,2

This section is devoted introducing a general notation to describe a 2 player game with complete information extended with the various communication phases. At different stages of the communication the players have different information concerning the past. Moreover, at every stage they learn some new information which depends on the message they have sent and whether the stage was mediated or not. After fixing the timing, we define the strategy space of the extended game. Finally, we are ready to state the main result,

Theorem 1. We do not define the extended game explicitly for 3 player games and for the games with incomplete information since all the results follow from Theorem 1. Though, when it is necessary we point out the important features of the corresponding strategies.

3.1   Notations for Theorem 1

Consider a general finite 2 player normal form game with complete information $\Gamma = (g, A)$, where $A = A^1 \times A^2$ the set of action-profiles and g^i is the payoff function for player $i \in \{1, 2\}$. We follow the notation of Gossner and Vieille (2001). $M^i = \{0, 1\}$ is the players' message space. To simplify the notation label the different input combinations as:

$h(m)$	0	1
0	a	c
1	b	$*$

The AND signaling function l^1 for player 1 can be described as $l^1(0, .) = \{a, c\}$ and $l^1(1, 0) = \{b\}$ and $l^1(1, 1) = \{*\}$. l^1 induces $\mathcal{P}^1 = \{\{a, c\}, \{b\}, \{*\}\}$ information partition. For player 2 symmetrically it is $\mathcal{P}^2 = \{\{a, b\}, \{c\}, \{*\}\}$. Let the players send messages $m_n = (m_n^1, m_n^2)$ simultaneously for $0 \le n \in \mathbb{N}$. Player i is told $l^i(m_n) \in \mathcal{P}^i$. The set of plays at time n is $H_n = \{a, b, c, *\}^n$ and denote $H_\infty = \{a, b, c, *\}^\mathbb{N}$. We denote $h(m_n)$ the play at stage n. Prior to sending the message in stage n the information available for player i is $\mathcal{H}_n^i$ an algebra generated by the cylinder sets of the form $h_n^i \times H_\infty$, where $h_n^i \in (\mathcal{P}^i)^n$ is a sequence of n elements. Denote $\mathcal{H}_\infty^i = \sigma(\mathcal{H}_n^i, n \ge 0)$ the σ-algebras over H_∞ generated by these σ-algebras. Define $\mathcal{H}_\infty$ similarly. Given a finite set E denote ΔE the set of probability distributions over the set E.

Fix $p \in \mathbb{N}$ and denote $_k m = (m_{(k-1)p}, \ldots, m_{kp-1})$ and $_k h = (h(m_{(k-1)p}), \ldots, h(m_{kp-1})) \in H_p$ for $k \ge 1$, that is the kth p-coordinates of a given play h. We call $_k h$ the kth *communication round* of length p and $_k m$ the corresponding messages during that round.

Say that the *communication was successful in the kth round*, if $_k h$ does not contain $\{*\}$.

Let $1 \le \kappa_1 < \ldots < \kappa_T$ denote random variables corresponding to the number of the first T successful rounds.

Assume that after $\kappa_T p$ stages of the mediated communication the players communicate directly. Let $t \ge 0$ denote the tth stage of the direct communication. The information partitions corresponding to the trivial signalling functions of the direct communication are $\mathcal{P}_d^i = \{\{a\}, \{b\}, \{c\}, \{*\}\}$ for $i = 1, 2$. The information available for player i at stage $\kappa_T p + t$ can be described by the σ-algebra $\mathcal{H}_{\kappa_T p}^i \otimes 2^{H_t}$.

3.2   The extended game

Now we fix the timing of the extended games and define their strategy-space. Consider the following timing:

1. the players communicate under the mechanism *AND* until T successful communication round of length p occur[12],

2. at stage $\kappa_T p$ start communicate directly and simultaneously $\log T + T + pT$ times repeatedly

3. finally choose an action in Γ.

Denote the extended game with $\Gamma_p(T)$. The strategy space of the extended game is:

1. the communication strategies through the *AND*: $\sigma^i = (\sigma_n^i)_{n \geq 0}$, where σ_n^i is $\mathcal{H}_n^i$-measurable mapping to ΔM^i.

2. the direct communication strategies: $\tau^i = (\tau_t^i)_{\log T + T + pT > t \geq 0}$, where τ_t^i is $\mathcal{H}_{\kappa_T p}^i \otimes 2^{H_t}$-measurable mapping to ΔM^i

3. decision rules ρ^i, $\mathcal{H}_{\kappa_T p}^i \otimes 2^{H_{\log T + T + pt}}$-measurable mapping to ΔA^i and ρ_∞^i, $\mathcal{H}_\infty^i$-measurable if there were no T successful rounds.

Denote $\pi = (\sigma, \tau, \rho)$ a strategy profile of the extended game. There is an induced distribution P_π on $(H_\infty \times H_{\log T + T + pt} \times A, \mathcal{H}_\infty \otimes \mathcal{H}_{\log T + T + pT} \otimes 2^A)$.

3.3 Theorem 1

We are ready to state the main result.

Definition 1 *Let $\epsilon > 0$. π is ϵ-Nash equilibrium of the extended game $\Gamma_p(T)$, if for any i and π'^i*

$$\mathbb{E}_{P_\pi} g^i(\mathbf{a}) + \epsilon \geq \mathbb{E}_{P_{\pi'^i, \pi^{-i}}} g^i(\mathbf{a}).$$

Definition 2 *An* information structure *on a set finite set A is a probability distribution μ over A. An element $a = (a^1, \dots, a^I) \in A$ is chosen with probability $\mu(a)$, then player i is informed of the component a^i.*

Definition 3 *An information structure μ on A is a correlated equilibrium of Γ if and only if*

$$\max_{a'^i} \mathbb{E}_{\mu(\mathbf{a}^{-i}|a^i)} g^i(a'^i, \mathbf{a}^{-i}) = \mathbb{E}_{\mu(\mathbf{a}^{-i}|a^i)} g^i(a^i, \mathbf{a}^{-i})$$

for all $i \in I$ and $a^i \in A^i$ where $\mu(a^i) > 0$.

Definition 4 *A correlated equilibrium of Γ is strictly individually rational (SIR) if and only if*

$$\mathbb{E}_{\mu(\mathbf{a})} g^i(\mathbf{a}) > \min_{a^{-i} \in \Delta(A^{-i})} \max_{a^i \in \Delta A^i} g^i(a^i, a^{-i})$$

for all $i \in I$.

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[12]If this event does not occur, the players communicate infinitely long and then play according to ρ_∞ see below.

Theorem 1 *If $I = \{1, 2\}$ then for any SIR correlated equilibrium μ of Γ there exists an extended game $\Gamma_p(T)$ and a π such that P_π is close to μ and π is an ϵ-Nash equilibrium of this extended game.*

To state the theorem formally, let us introduce a distance function on ΔE as $d(\mu, \nu) = \max_{e \in E} |\mu(e) - \nu(e)|$.

Then for any $\epsilon > 0$ and $\delta > 0$, for any finite $\Gamma = (g, A)$ and for any $\mu \in \Delta A$ such that μ is a correlated equilibrium of Γ, there is $\Gamma_p(T)$ and a π such that $d(P_\pi, \mu) \leq \delta$ and π is an $\epsilon-$Nash equilibrium of the extended game.

Remark 1 ***Transparency, secrecy***: *The equilibrium strategies can be constructed in a way that:*

1. *the players can detect with arbitrary high probability if the communication device does not calculate the public announcements according to the AND function,*

2. *after the communication, the AND mediator has NO knowledge at all about the players action in Γ.*

The remark is obvious, because of the followings. First, before engaging in the mediated communication the players can privately agree on some randomly chosen stages where they test the mediator. If both players send at those pre-agreed stages the message 1 and the mediator is announcing 0, the mediator is caught out in a lie. Notice however, in case of deviation, it is not clear that one of the players or the mediator was deviating. Namely one of the players can send 0 and blame the public message 0 on the mediator's deviation and vice versa. That is why we need the secret eavesdropping for the generalization of Theorem 1.

Second, the players can just simply send completely random messages at some pre-agreed stages of the mediated communication phase. In this way, players can mask their relevant communication. The mediator cannot distinguish the random noise, introduced by the players from the messages on which the players condition their actions. So the mediator gets no information at all.

3.4 Theorem 2

Theorem 2 states that theorem 1 can be extended to games with 3 players. The intuition is that one of the players can play the role of the mediator, and serve the other players with correlated private information in a way that the mediating player gets no information about the outcome of the correlation. Any possible deviations can be detected with arbitrary high probability and the deviator can be identified and punished on his minmax level.

We have to define an extended game $\Gamma^c = (P, \Gamma)$. P lasts T stages of communication,

where in each stage each player i is able to make a public announcement and can send send private messages to players $-i$ from a message set $M^i = \mathbb{N}$, and before sending a private message each player can decide Bcc another player to eavesdrop the message he is about to send to the third player. Then players choose an action in Γ. Clearly a strategy profile of Γ^c induces a distribution in Γ.

The signalling structures of the private and public messages are trivial. We want to clarify what is happening in case of eavesdropping. Before each stage of communication for example player 1 has the possibility to "invite" player 2 to be the witness of the private message 1 is about to send to player 3. The invitation can be thought of as 1 using a fake private channel for his communication with 3. 3 never knows that 2 got to know the message that he received from 1 or not. If 2 is eavesdropping the channel, 1 cannot send any message to 3 without 2 getting to know the message. It is like 1 makes a public announcement though 3 does not know if 2 is present or not. For a correct formulation of the signalling structure of such messages see Billot, Vergnaud and Walliser (2005).

Possibly the easiest way to formulate such a situation if each message m has a copy m'. If m is sent to a player, then players different from the sender and the receiver have no knowledge about the content of the message. The receiver though does not know whether he received m or m'. m was sent privately. However if m' was sent, then a player different from the sender and the receiver knows that m' was sent. The receiver though does not know whether he received m or m'.

Theorem 2 *If $|I| = 3$ then for any correlated equilibrium μ of Γ with rational entries, there exists an extended game Γ^c and a Nash equilibrium strategy profile such that the induced distribution is μ.*

4 From Correlated to Communication equilibria

In the previous section we have stated that, it is possible to build any correlation device. It is clear now that if we have an arbitrary two person game Γ then any correlated equilibrium of Γ can be achieved as a Nash equilibrium of an extended game where:

1. two players are allowed to communicate repeatedly through the mediator *AND*,

2. after the *mediated* communication endogenously terminates, the players engaged in a *direct* repeated communication phase where the messages are sent simultaneously,

3. finally the players choose a *strategy* in Γ.

To maintain the Nash equilibrium it is crucial that the players have a punishment strategy in case of deviation, namely minmax-ing the other player.

It turns out that the construction above applies to Bayesian games and to the set of communication equilibria as well with 2 modifications:

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1. obviously, the punishment strategies has to be different,

2. after the correlation phase the players still need to communicate with each other.

To be precise let us formulate a Bayesian game and define its set of communication equilibria. We also define the appropriate punishment strategies. In Theorem 3 we state that the communication equilibrium basically can be traced back generating correlations that is correlated equilibria if after the correlation phase and having received their information the players can still communicate. Finally as the Corollary of Theorem 1 and Theorem 3 we can obtain communication equilibria as Bayesian Nash equilibria of an extended game with mediated and unmediated communication.

Let $\Gamma =< I, L^i, A^i, g^i, \lambda, i \in I >$ be a Bayesian game, where I is the set of players, L^i is the set of possible types of player i, A^i is the set of actions for player i. Let $L = \times_{i=1}^{I} L^i$ and $A = \times_{i=1}^{I} A^i$, and $g^i : L \times A \to \mathbb{R}$ the payoff function of player i and $\lambda \in \Delta L$.

A Bayesian Nash equilibrium of Γ is a strategy profile $s = (s^1, \dots, s^I)$ where $s^i : L^i \to \Delta A^i$ are such that:
$$s^i(l^i) \in arg \max_{\Delta A^i} \sum_{l^{-i} \in L^{-i}} \lambda(l^{-i}|l^i) g^i(., s^{-i}(l^{-i}), (l^i, l^{-i})).$$
for all $l^i \in L^i, i \in I$.

Let $q : L \to \Delta A$ and think of the following extended game Γ^q:

1. players learn their types say $l \in L$,

2. players can send private message from L^i to a mediator, say the sent message profile is l',

3. the mediator chooses an action profile $a \in A$ with probability $q(l')(a)$

4. the mediator sends a^i privately for player i,

5. players choose an action in Γ.

Definition 5 *q is a communication equilibrium of* Γ *if and only if:*
$$g^i[q|l^i] \geq \sum_{l^{-i}} \lambda(l^{-i}|l^i) \sum_{a} q(l'^i, l^{-i})(a) g^i(r(a^i), a^{-i}, (l^i, l^{-i}))$$
for all i, l^i, l'^i *and for all* $r : A^i \to A^i$*, where*
$$g^i[q|l^i] = \sum_{l^{-i}} \lambda(l^{-i}|l^i) \sum_{a} q(l^i, l^{-i})(a) g^i(a, (l^i, l^{-i})).$$

is the expected payoff of player i *of type* l^i *when all the players send their true types and play according to the suggestion of the mediator*[13]*.*

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[13]Notice that if L is a singleton (the case of complete information), then q is a communication equilibrium if and only if it is a correlated equilibrium of Γ.

The expected payoff of a strategy profile s for player i of type l^i can be written as:

$$g^i[s|l^i] = \sum_{l^{-i}} \lambda(l^{-i}|l^i) \sum_a s(l^i, l^{-i})(a) g^i(a, (l^i, l^{-i})).$$

Definition 6 *q communication equilibrium Nash dominant for player $i \in I$ if there is an s Bayesian Nash equilibrium such that for all $l^i \in L^i$,*

$$g^i[s|l^i] < g^i[q|l^i].$$

q is Nash-dominant if it is Nash-dominant for all $i \in I$.

Notice that, the dominated equilibrium can vary across the players. Unlike in Ben-Porath (2003), s has to be the same for all the players. We call such a q *strongly Nash-dominant.*

Theorem 3 *Every q Nash-dominant communication equilibrium payoff of Γ is a correlated equilibrium payoff of an extended game, where after having received their information as in Γ, the players can communicate directly.*

Now consider a Bayesian game Γ and one of its communication equilibria q which is Nash-dominant. Take the following Bayesian game Γ^d which is an extension of Γ:

1. the players learn their types,

2. communicate repeatedly through the *AND* communication device,

3. plain communication,

4. choose an action in Γ.

Then we have the following corollary of Theorem 1 and Theorem 3:

Corollary 1 *An $\epsilon-$Bayesian Nash equilibrium of Γ^d induces q of Γ.*

Now let $|I| = 3$ and q be a Nash-dominant communication equilibrium with rational entries. Then as a corollary of Theorem 2 and Forges (1990).

Corollary 2 *A Bayesian equilibrium of the game extended with cheap talk with Bcc induces q of Γ.*

5 Proof of Theorem 1

We fix a game Γ and one of its correlated equilibrium distributions μ. We construct a strategy-profile π and then show that it generates a distribution P_π δ-close to μ. Finally we show that π is an ϵ-Nash equilibrium and discuss the remarks.

We follow the proof through the example introduced in section 2, take the following distribution μ:

	0	1
0	2/5	1/5
1	1/5	1/5

Fix a Γ and a $\mu \in \Delta A$ such that μ is a correlated equilibrium of Γ. Fix $\epsilon > 0$ and $\delta > 0$.

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5.1 π

In this section we describe a strategy-profile π. We proceed step by step. First we define σ, then τ and finally ρ.

5.1.1 Lehrer's protocol by the *AND*, the σ

During the mediated phase the players communicate according to σ, which builds up by independent identical repetitions of Lehrer's communication strategy σ_L. To describe Lehrer's protocol we need to introduce an auxiliary table: Number the 0s in the table

	1	2	3	4
1	1/10	1/10	1/10	0(0)
2	1/10	1/10	0(1)	1/10
3	1/10	0(2)	1/10	0(3)
4	0(4)	1/10	0(5)	1/10

Figure 9: Auxiliary table.

starting with 0 from the top to the bottom from the left to the right. For example the 0 in the third row fourth column has number 3. There are 6 zeros so Lehrer sets $p = 6$ and define $\sigma_L = (\sigma_0, \ldots, \sigma_{p-1})$ as follows. The players randomize over 4 different message sequences of length 6 with equal probabilities $\frac{1}{4}$. The interpretation of these sequences is that the row

	m_0^1	m_1^1	m_2^1	m_3^1	m_4^1	m_5^1
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0	0	1	1	0	0
4	0	0	0	0	1	1

Figure 10: The 4 different 6 stage long message sequences of the row player.

and the column player communicates according to the row and column she has chosen in the auxiliary table. The players are basically answering yes (1) no (0) questions concerning the 0s in the table Figure 9. For example if the row player communicates according to the row 3, she is answering with yes (sends message 1 at stage 2 and 3 and message 0 otherwise) to the 0s numbered 2 and 3. By choosing one of the rows and columns in the

	m_0^2	m_1^2	m_2^2	m_3^2	m_4^2	m_5^2
1	0	0	0	0	1	0
2	0	0	1	0	0	0
3	0	1	0	0	0	1
4	1	0	0	1	0	0

Figure 11: The 4 different 6 stage long message sequences of the column player.

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auxiliary table in Figure 9., the players in fact choose "intended" actions. This is given by the following function β for the row player, player 1, and for the column player, player 2:

$$\beta^1(1) = \beta^1(2) = \beta^2(1) = \beta^2(2) = 0, \beta^1(3) = \beta^1(4) = \beta^2(3) = \beta^2(4) = 1.$$

Lehrer's protocol can be describe as follows:

1. Each player chooses a number 1,2,3,4 with probability $\frac{1}{4}$,

2. the players communicate for 6 stages according to the chosen number that is, according to the corresponding row and column in the auxiliary table,

3. if the mediator did not make the public announcement 1 that is, when the communication round of length 6 was successful, the players play according to β,

4. if there was a public announcement 1, the players repeat the procedure that is, select a number from 1,2,3,4.

Formally, let $\beta^i : H_p \to A^i$ interpretations $\mathcal{H}_p^i$-measurable mappings. We call $\pi_L = (\sigma_L, \beta)$ an L-protocol of length p, where $\sigma_L^i = (\sigma_0^i, \ldots, \sigma_l^i, \ldots, \sigma_{p-1}^i)$ $\mathcal{H}_l^i$-measurable mappings, communication strategy under the mechanism AND. Clearly there is an induced distribution P_{π_L} by π_L on $(H_p \times A, \mathcal{H}_p \otimes 2^A)$. Denote its marginal on H_p by P_{σ_L}. Denote $S \subset H_p$ the set of plays which contain no $\{*\}$, that is the set of successful rounds. Note that $S \in \mathcal{H}_p^i$ for all i.

Proposition 1 *Lehrer(1991) (JCC): For any finite A and $\mu \in \Delta A$ there is a p and π_L L-protocol, such that $\forall a \in A$ and for all i:*

1.
$$P_{\pi_L}(\mathbf{a} = a|S) = \mu(\mathbf{a} = a),$$

2.
$$P_{\pi_L}(\mathbf{a}^{-i} = .|\mathbf{a}^i, S) = P_{\pi_L}(\mathbf{a}^{-i} = .|\mathcal{H}_p^i \cap S),$$

3.
$$P_{\sigma_L}(h_p|\mathcal{H}_p^i \cap S^c) = 1 \text{ or } 0.$$

The first property states that, conditional on the event that the players have not got a $\{*\}$, the induced conditional distribution on A is exactly μ.

This can be seen in the example, if one aggregates the columns and rows of Figure 8. according to β.

The second property states that, on the sub$-\sigma-$algebra, where there is no $\{*\}$, $\mathbf{a}^1$ is a sufficient statistic for $\mathbf{a}^2$ under P_{π_L}. This means that when a player learns her h_p^i, her information about the other player's action is $\mu(\mathbf{a}^{-i}|\beta^i(h_p^i), S)$, that is exactly the same as she would have learnt her action.

Assume that the row player was communicating according to row 3 and the column player according to column 1 in the auxiliary table. That is the sent messages were:

$$R : 0, 0, 1, 1, 0, 0$$

$$C : 0, 0, 0, 0, 1, 0$$

and the resulting public announcements correspondingly:

$$0, 0, 0, 0, 0, 0.$$

The row player learns that the column player was not choosing the second and the fourth column in the auxiliary table, so she believes that the column player plays with probability 0.5,0.5 her action 0 or 1. This is exactly $\mu(\mathbf{a}^2|\beta^1(3))$. Here $\beta^1(3)$ refers to $\beta^1(h_p^1)$, where h_p^1 is the information of player 1 if she plays according to row 3. That is no matter what h_p^1 is player 1 plays according to her communication strategy, which we abbreviated by 3 that is, with the row she was chosen in the auxiliary table.
The column player learns that the row player was not choosing the fourth row in the auxiliary table, so she believes that the row player plays with probability $\frac{2}{3}$ her action 0 and $\frac{1}{3}$ her action 1. This is exactly $\mu(\mathbf{a}^1|\beta^2(1))$.

The third property says, that whenever players get a $\{*\}$ they know P_{σ_L} almost sure the realized play.

As we pointed out in the introductory example, Lehrer's protocol is manipulable. For example player 1 is better of by choosing always 3 or 4 and communicate accordingly and after a successful communication round she can play action 1. However if the players do not play after the first successful round, but they repeat the communication according to σ_L sufficiently many times, these kind of deviations are statistically detectable.

For π, define the communication strategies under the mechanism AND by:

$$\sigma = \Pi_{k \in \mathbb{N}} \sigma_L,$$

that is, σ builds up of independent identical repetitions of σ_L.

5.1.2 Direct Talk, the τ

After the mediated communication, the players send direct messages simultaneously for $\log T + T + pT$ stages. In the first $\log T$ stages the players conduct a jointly controlled lottery which allows them to mark and coordinate on one of the successful rounds.

In the last pT stages called the reporting phase, the players reveal their past messages corresponding to the successful rounds of the mediated communication phase, excluding the round picked by the jointly controlled lottery. This phase is needed to avoid statistically undetectable deviations, such as spying strategies, in the mediated phase. The

additional T stages is needed to decided by a joint lottery which player is reporting in the corresponding round.

Let $h_{\kappa_T p}$ be the realized play and $1 \leq \kappa_1 < \ldots < \kappa_T$ denote the first T successful rounds, that is $_{\kappa_s} h \in S$ for $s \in \{1, \ldots, T\}$.

<center>Jointly Controlled Lottery, κ^*</center>

The players want to coordinate on one of the successful rounds and play an action according to their "intended" action β in that round.

Define the first $\log T + T$ coordinates of τ^i as $\tau_t^i(.)(0) = \tau_t^i(.)(1) = 0.5$ for $t \in \{0, \ldots, \log T + T - 1\}$ whatever the communication history was so far, that is τ^i randomizes uniformly on M^i at first $\log T + T$ stages independently, no matter what happened in the past. Let $(m_0, \ldots, m_{\kappa_T p + \log T + T - 1})$ denote the sent messages so far and $(h_{\kappa_T p}, h_{\log T}, h_T)$ the corresponding play. Let $f : \{0, 1\} \times \{0, 1\} \to \{0, 1\}$ be the following function:

f	0	1
0	1	0
1	0	1

or for some $h(m)$:

h	a	b	c	*
f	1	0	0	1

With abuse of notation define $f(h_{\log T}) = 1 + \sum_{r=0}^{\log T - 1} f(h(m_{\kappa_T p + r})) 2^r$.

Proposition 2 *Aumann et. al. (1995)(Jointly controlled lotteries): Such an f function by the first $\log T$ coordinates of τ induces a uniform distribution on $\{1, \ldots, T\}$ in a way that no unilateral deviation from τ affects this distribution.*

Intuitively, the first $\log T$ stage of the direct talk allows the players to choose jointly and uniformly one of the successful rounds of the mediated communication phase. Denote

$$\kappa^* = \kappa_{f(h_{\log T})}$$

the jointly chosen successful round. If players then play according to

$$\beta(_{\kappa^*} h)$$

their "intended" action in the corresponding successful round, then the protocol induces the same distribution and same information structure as that of Lehrer.

<center>Reporting phase</center>

In this phase of the direct talk the players reveal all their past messages they have sent in the mediated rounds excluding those from the one selected by the jointly controlled lottery κ^*. This phase can rule out otherwise non-detectable deviations in the mediated phase.

We show an example for a non-detectable deviation during the mediated phase. The row player can randomize with 0.25,0.25 communicating according to the first and second row of the auxiliary table and choose a "spying" strategy otherwise. The row player can send message 1 when the question is about the 0(2) and 0(4) in the half of the time remained. Doing so the row player can get the knowledge that the column player is going to play her action 1. Then in another round the row player can send message 1 when the question is about the 0(3) and 0(5) in the other half of the remained time. Doing so the row player can get the knowledge that the column player is going to play her action 0.

This kind of deviation cannot be detected statistically because it induces the same distribution μ, however the row player's information is more than needed for the information structure μ. Notice also that this deviation is not just a shifting of the probabilities in the randomization σ_L^1 but uses communication strategies which are outside of the support σ_L^1.

If in the reporting phase the players have to reveal their past messages, then a spying player has to lie about her sent messages. These lies can be detected with positive probability. For example when the row player was spying by sending 1 for 0(2) and 0(4), she has to lie but she does not know[14] that the column player was communicating according to column 3 or 4. Then it can be the case, that the column player gets a contradictory report from the row player.

Up to time $\kappa_T p + \log T + T$ the communication strategies $\sigma, (\tau_t)_{t \in \{0, \ldots, \log T + T - 1\}}$ of the players were completely[15] history independent. From $(\tau_{\log_T + T})$ on the communication strategies depend on the lottery on h_T and on the sent messages in the mediated communication phase.

Let $R^i(h_T) = \{1 \leq s \leq T | f(m_{\kappa_T p + \log T + s - 1}) + 1 = i\}$. $R^i(h_T)$ is a randomly selected subset of $\{1, \ldots, T\}$. The randomness comes through h_T which is jointly chosen by the players through $(\tau_{\log_T + s - 1})_{s=1,\ldots,T}$ defined above. $R^i(h_T)$ will have the interpretation that tells to player i that she has to report her mediated messages corresponding to the successful rounds $\kappa_s \neq \kappa^*$ for $s \in R^i(h_T)$. Whenever player i is reporting player $-i$ sends random messages.

Define $\tau_{T + \log T + (k-1)p + l}^i(h^i(m_{(k-1)p + l}))(m_{(k-1)p + l}^i) = 1$ for $l = 0, \ldots, p-1$ if and only if $k \neq \kappa^*$ and $k = \kappa_s$ where $s \in R^i(h_T)$, otherwise set $\tau_t^i(.)(0) = \tau_t^i(.)(1) = 0.5$ for $t \geq T + \log T$.

In words, player i sends the messages he has sent in the stages of the successful medi-

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[14]It is also possible the a player sends 3 times the message 1 and figures it out exactly what the other player have sent. Though in these cases he modifies the induced distribution, which will be detected statistically by the repetition of the protocol.

[15]As σ is defined.

ated communication rounds corresponding to the set $R^i(h_T)$, but i randomizes when the corresponding round is about the successful round κ^* chosen by the jointly controlled lottery and in the stages corresponding to rounds where $-i$ reports that is in $R^{-i}(h_T)$. We refer to such a direct communication strategy in the reporting phase as *true reporting*.

At the end of the unmediated communication at time $p\kappa_T + \log T + T + pT$ the players face a play $h = (h_{\kappa_T p}, h_{\log T}, h_T, h_{pT})$ given their information structures $\mathcal{H}^i_{\kappa_T p} \otimes 2^{H_{\log T + T + pT}}$. To simplify the indexing let us denote $_sm' = (m_{\kappa_T p + \log T + T + (s-1)p}, \ldots, m_{\kappa_T p + \log T + T + sp - 1})$.

Notice that under a true reporting strategy $_{\kappa_s}m^i = \,_sm'^i$ for $s \in R^i(h_T)$ but $s \neq f(h_{\log T})$. We say that a lie was detected by player i in the reporting phase if and only if $_sm'^{-i}$ contradicts with $_{\kappa_s}h^i$ for some $s \neq f(h_{\log T}) \in R^{-i}(h_T)$. Formally when:

$$P_{\sigma_L}\big(_sm'^{-i} \mid \,_{\kappa_s}h^i\big) = 0$$

Which means that player i has sent $_{\kappa_s}m^i$ in round κ_s of the mediated communication phase but player $-i$ has sent $_sm'^{-i} \neq_{\kappa_s} m^{-i}$ in the reporting phase, when she had to send $_{\kappa_s}m^{-i}$ what she has sent in fact in the mediated phase. So if $_sm'^{-i}$ is incompatible with $_{\kappa_s}h^i$ according to σ_L then player $-i$ was caught out in a lie.

Let L^i be the set of histories, where no lie was detected by player i.

5.1.3 Decision rules, ρ

In this section we describe 2 statistical tests performed by the players on the realized play of communication. The first one is calculated using the information of the mediated phase, while the second according to the reporting phase. Then players decided according to the outcome of the test whether to play according to $\beta(_{\kappa^*}h)$ or punish a possible deviation.

Statistical tests, $C(\gamma)$

Now assume that κ_T repetitions of the mediated communication rounds of length p have been made and there were T successful rounds. After the direct communication set $h = (h_{\kappa_T p}, h_{\log T}, h_T, h_{pT})$ as the realized play.

By the third property of Lehrer's protocol player i can calculate the following for all $h_p \notin S$:

$$\hat{P}^{*i}_h(h_p) = \frac{\sum_{k=1, _kh^i \notin S}^{\kappa_T} P_{\sigma_L}(h_p \mid \,_kh^i)}{\kappa_T - T}$$

Notice that $\hat{P}^{*i}_h = \hat{P}^{*-i}_h \approx P_{\sigma_L}(h_p \mid S^c)$ if players follow σ.

After the reporting phase the players perform a statistical test on the revealed messages. If the empirical frequency of the revealed messages is close to the distribution induced by σ_L

the players accept the hypothesis that the other player was communicating according to σ_L.

For an $h_p \in S$ let $I_{h_p}^i(\,_s m'^{-i}) = 1$ if $h(_s m'^{-i},_{\kappa_s} m^i) = h_p$ and 0 otherwise. That is I_{h_p} counts the number of h_p according to the reporting phase. Set

$$\hat{P}_h^i(h_p) = \frac{\sum_{s \in R^{-i}(h_T), \kappa_s \neq \kappa^*}^{\kappa_T} I_{h_p}(\,_k m'^{-i})}{|R^{-i}(h_T)| - 1}$$

to be the empirical relative frequency of h_p in the report h'_{pT} when player $-i$ was reporting. This frequency, under the true reporting strategy, corresponds to the distribution $P_{\sigma_L}(h_p|S)$.

Let us set player i's confidence set for some γ as follows.

$$C^i(\gamma) = \{h|d(\hat{P}_h^{*i}, P_{\sigma_L}(.|S^c)) < \gamma \text{ and } d(\hat{P}_h^i, P_{\sigma_L}(.|S)) < \gamma\}$$

That is in C^i player i accepts the hypothesis that, in fact player $-i$ played according to σ^{-i}.

$$\rho$$

Given Γ let $x^i \in \Delta A^i$ the punishing strategy of player i against player j, that is:

$$x^i = arg \min_{y^i \in \Delta A^i} \max_{y^j \in \Delta A^j} g^j(y^i, y^j).$$

Define the decision rules as follows for $h = (h_{\kappa_T p}, h_{\log T}, h_T, h_{pT})$:

$$\rho^i(h) = \beta^i(\,_{\kappa_{f(h_{\log T})}} h^i).$$

for $h \in L^i \cap C^i(\gamma)$ and x^i otherwise.

In words, player i plays according to the interpretation β^i in π_L respecting the $s = f(h_{\log T})$th successful round of the mediated communication (that is the $\kappa^* = \kappa_s$ round) if he accepted the hypothesis that player j was communicating according to σ^j and j was not caught out in a lie in the report phase. Otherwise player i punishes j with x^i.

Proposition 3 π *is well defined.*

Proof: It is plain by the construction. ∎

5.2 Close to μ for the δ

In this section we prove that the induced distribution can be close to μ if the mediated communication phase is long enough. Intuitively, by the law of large numbers the realized history fall into $C(\gamma) = C^1 \cap C^2$ and so players play according to β. Thus the induced distribution is that of P_{π_L} which by construction equals μ.

Lemma 1 *For any γ, there is a $T(\gamma)$ such that $d(P_\pi, \mu) < \delta$*

Proof: It is clear that conditional on C the players have expectations $P_\pi(a|C) = \mu(a)$. This follows by Proposition 1 and 2 and by the construction. That is due to the independent repetitions of σ_L and the uniform selection from the successful rounds by τ. Also

$$P_\pi(a) = P_\sigma(C)P_\pi(a|C) + (1 - P_\sigma(C))P_\pi(a|C^c),$$

thus if we can make $P_\sigma(C)$ close to 1, than the P_π will be close to μ. Notice that κ_T is a negative binomial random variable with success probability $P_{\sigma_L}(S) < 1$ and parameter T. Thus as T increases $\kappa_T - T$ increases as well. Also $|R^{-i}(h_T)|$ increases by T, so for T big enough by the weak law of large numbers for independent identically distributed random variables (Feller (1971)):

$$P_\sigma(C) > 1 - \delta_4$$

for any δ_4. In words, both players will accept their hypothesis with arbitrary high $(1 - \delta_4)$ probabilities even if γ is small. Then

$$d(P_\pi, \mu) < \delta$$

for any δ if δ_4 is small enough and $T(\gamma)$ is big enough for any $\gamma > 0$. $\blacksquare$

5.3 Deviations for the ϵ

In this part of the proof we show that none of the players can gain more than ϵ by deviating from π. Now we have to check that:

$$\epsilon \geq \mathbb{E}_{P_{\pi'2,\pi^1}} g^2(a) - \mathbb{E}_{P_\pi} g^2(a).$$

for all π'^2. For the proof we also use the following proposition[16] of Lehrer (1991). Denote:

$$Z = \{h_p \in S | P_{\pi_L}(h_p) = 0\}.$$

the set of plays of length p not containing $\{*\}$ and having 0 probability under P_{π_L}.

Proposition 4 *Lehrer (1991): For any $\delta_4 > 0$ there is a $\gamma(\delta_4)$ and $T(\gamma)$ such that for any i and $\pi'^i = (\sigma'^i, \tau'^i, \rho'^i)$ for which $P_{((\sigma'^i,\tau'^i),(\sigma^{-i},\tau^{-i}))}(L^{-i} \cap C^{-i}(\gamma)) \geq \delta_4$ the following hold:*

1.
$$P_{(\sigma'^i,\tau'^i),(\sigma^{-i},\tau^{-i})}\left({}_{\kappa^*}\mathbf{h} \in Z | L \cap C\right) < \delta_4,$$

2.
$$d(P_{(\sigma'^i,\tau'^i),(\sigma^{-i},\tau^{-i})}\left({}_{\kappa^*}\mathbf{h} | L \cap C\right), P_{\sigma_L}(\mathbf{h}_p|S)) < \delta_4'$$

---

[16]Thanks to Olivier Gossner for pointing out the connection with his proof in Gossner (1995).

In words, if player i wants to pass the tests and do not want to be caught out in lie with probability at least δ_4, then he has to conduct a communication strategy (σ'^i, τ'^i) which conditional on passing the tests,

1. $_{\kappa^*}h$ will be out of the support of P_{σ_L} with probability less than δ_4,

2. generates a distribution δ'_4 close to $P_{\sigma_L}(|S)$.

We need an equivalent definition of the correlated equilibrium:

Lemma 2 $\mu \in \Delta A$ is a correlated equilibrium of G, if and only if the following condition holds for all $r : A^j \to A^j$ and symmetrically for i:

$$\mathbb{E}_\mu g^j(a) \geq \mathbb{E}_\mu g^j(a^i, r(a^j))$$

Proof: Straightforward calculation[17]. ∎

We set an appropriate γ for the confidence set and a $T(\gamma)$ to maintain the ϵ-constraint.

Proposition 5 There is a γ and $T(\gamma)$ such that for any deviation $\pi'^2 = (\sigma'^2, \tau'^2, \rho'^2)$:

$$\epsilon \geq \mathbb{E}_{P_{\pi'^2,\pi^1}} g^2(a) - \mathbb{E}_{P_\pi} g^2(a).$$

Proof: Set $D^j = \sum_{a \in A} g^j(a)$ and $D = \max_{j \in \{1,2\}} D^j$. By proposition 4, for any δ_4 there is a γ and a $T(\gamma)$ such that for any deviation $\pi'^2 = (\sigma'^2, \tau'^2, \rho'^2)$:

$$\max_{\pi'^2} \mathbb{E}_{P_{\pi'^2,\pi^1}} g^2(a) \leq \max_{\nu, r} \mathbb{E}_\nu g^2(a^1, r(a^2)) + \delta_4 D \qquad (1)$$

where $r : A^2 \to A^2$ and $\nu \in \Delta A$ such that $d(\nu, \mu) \leq \delta_4$.

Suppose that π'^2 passes the test with probability more than δ_4. Then the first term can be obtained by choosing δ'_4 sufficiently small by property 2 of Proposition 4 and property 1 of Proposition 1. The second term: if player 2 passes the test she can gain at most D if she is in Z with probability less than δ_4 by property 1 of Proposition 4. On the other hand if π'^2 does not pass the test with probability more than δ_4 the inequality is straightforward.

Set γ and $T(\gamma)$ such that (1) holds for $\delta_4 < \frac{\epsilon}{3D}$ and $d(P_\pi, \mu) \leq \delta_4$ as well. Then

$$\mathbb{E}_\mu g^2(a^1, r(a^2)) \leq \mathbb{E}_\mu g^2(a)$$

for any r since μ is a CE. By simple calculation

$$\mathbb{E}_{P_\pi} g^2(a) + \delta_4 D \geq \mathbb{E}_\mu g^2(a).$$

---

[17]In fact the proof could be done without r. Player j, the deviating player will always want to play according to $a^j = \beta^j(_{\kappa^*}h^j)$ if he played in the support of σ_L^j in the round κ^*, since player i is mixing according to σ_L^i.

Then we have

$$\max_{\nu, r} \mathbb{E}_\nu g^2(a^1, r(a^2)) - \mathbb{E}_\mu g^2(a) \leq \mathbb{E}_{\nu^*} g^2(a^1, r^*(a^2)) - \mathbb{E}_\mu g^2(a^1, r^*(a^2)) \leq \delta_4 D,$$

where ν^*, r^* are the $arg \max$, where $d(\nu^*, \mu) \leq \delta_4$. Finally:

$$\max_{\pi'^2} \mathbb{E}_{P_{\pi'^2, \pi^1}} g^2(a) - (\mathbb{E}_{P_\pi} g^2(a) + \delta_4 D) \leq \mathbb{E}_{\nu^*} g^2(a^1, r^*(a^2)) + \delta_4 D - \mathbb{E}_\mu g^2(a),$$

and so

$$\mathbb{E}_{P_{\pi'^2, \pi^1}} g^2(a) - \mathbb{E}_{P_\pi} g^2(a) \leq \frac{\epsilon}{3} + \frac{\epsilon}{3} + \frac{\epsilon}{3}.$$

The same is true for player 1. ■ Q.E.D.

6 Proof of Theorem 3

Fix a Bayesian game Γ and a communication equilibrium q such that it is $s, (s')$-dominant for player i and $-i$ respectively. To prove the theorem we have to construct a correlated equilibrium of an extended Bayesian game, where in the interim stage players can communicate arbitrarily but directly. The support of the correlated equilibrium will depend on Γ and its distribution on q. First we explain the functioning of the correlation device (CD), second the way the two player communicate with each other. Finally we point out the stage where possible deviation can happen and show the way it can be avoided.

The CD selects:

1. randomly a permutation of the elements of L^i say η^i for each i independently. Let $\eta = (\eta^1, \eta^2)$.

2. $(a_{\eta(l)})_{l \in L}$ according to $q(l)$,

3. randomly permutations of the elements of A^i for each $l \in L$ and for each i independently say $(\phi_l^i)_{l \in L}$.

The CD sends $\eta^i, (\phi_{\eta(l)}^{-i}(a_{\eta(l)}^{-i}))_{l \in L}, (\phi_{\eta(l)}^i)_{l \in L}$ to player i.

Up to this point players do not have any additional information.

Let for all i, player i of type l^i announce $\eta^i(l^i)$. Now for all i, player $-i$ sends $\phi_{\eta(l)}^i(a_{\eta(l)}^i)$ to player i and so i can compute $a_{\eta(l)}^i = \phi_{\eta(l)}^{i-1}(\phi_{\eta(l)}^i(a_{\eta(l)}^i))$ and take this action. $(a_{\eta(l)}^i, a_{\eta(l)}^{-i})$ was chosen according to $q(l)$.

Player i of type l^i cannot be better of by sending a message different from $\eta^i(l^i)$. His information all along the communication process is the same as in the communication equilibrium q. The only possible deviation of $-i$ is sending a different action than $\phi_{\eta(l)}^i(a_{\eta(l)}^i)$

for player i. In this way i may compute a different action than $a^i_{\eta(l)}$. But this deviation can be avoided in the same way as in Forges (1990) in the 3 player case.

Let the CD randomly select from a set C^i codes for all $(l, a^i) \in L \times A^i$ that is, $c^i(\eta(l), a^i) \in C^i$ and send it to player i. The CD sends to player $-i$ only just the codes corresponding to the pairs $(\eta(l), \phi^i_{\eta(l)}(a^i_{\eta(l)}))$ that is, the codes $(c^i(\eta(l), \phi^i_{\eta(l)}(a^i_{\eta(l)})))_{l \in L}$. Player $-i$ has to send the code together with the permuted action and i respectively. These messages need to be sent simultaneously[18]. Now player i after the announcement of $\eta(l)$ waits for some $b^i \in A^i$ and the correct code $c^i(\eta(l), b^i)$, but the only code connected to $\eta(l)$ which $-i$ knows is $c^i(\eta(l), \phi^i_{\eta(l)}(a^i_{\eta(l)}))$. If player $-i$ sends some b^i with a wrong code, then he was caught out in a lie. In this case i plays s'^i, his part of the Bayesian Nash equilibrium s' which is dominated by q for player $-i$. If C^i is large enough, player $-i$ fails with high probability if he deviates and gets punished. In case of any other unusual message the players can turn to play s or s' implicitly.Q.E.D.

7   Proof of Theorem 2

First we show, how to generate any information structure with rational entries for players 1 and 2, in a way that player 3 has no information about the outcome of the correlation, yet players can deviate.

Second we show, how to immunize this protocol against deviations in a way that the identity of the deviator is revealed with high probability. Here we use the possibility of Bcc messages.

At this point we have a protocol which generates any information structure for player 1 and 2 and player 3 does not know anything about the outcome. Moreover the protocol reveals any deviation and the identity of the deviator with arbitrary high probability.

Finally we show how can such a protocol be used to generate any information structure with rational entries for player 1,2 and 3. The basic idea is that player 1 and 2 jointly chooses an action of player 3. This choice can be done in a way that 1 and 2 do not get additional information about the chosen action of player 3. Then the situation is that player 1 and 2 faces an incomplete information game where the states of the world are player 3's possible actions. Then we can apply the idea in the proof of Theorem 3 with a slight modification.

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[18]Otherwise a player could learn her action and decide to deviate or not. In this case the punishment strategies may not be effective.

7.1 P3(1,2)

We describe a protocol for a given order of the players. Then we need to change the role of player 1 and 2.

We have a set $E_3 = E^1 \times E^2$ and $\mu^3 \in \Delta E_3$ an information structure for player 1 and 2. Let for some set X, $I : X^d \to \Delta X$ calculate de relative frequencies in a $d-$long sequence $(x_1, \ldots, x_d) \in X^d$. Let d be such that the following is meaningful: $E^i(\mu^3) = \{(e_1^i, \ldots, e_d^i) \mid I(e_1^i, \ldots, e_d^i) = \mu^3(\mathbf{e}^i)\}$, that is the $d-$long sequences with elements from E^i such that the relative frequencies are equal to the marginal distribution of μ^3 on E^i. Let player 1 choose a sequence of $(e_1^1, \ldots, e_d^1) \in E^1(\mu^3)$ randomly and send it privately to 3. There is a correspondance $f_1^3 : E^1(\mu^3) \to P(E^2(\mu^3))$ such that for any $(e_1^2, \ldots, e_d^2) \in f_1^3(e_1^1, \ldots, e_d^1)$ it is true that $I((e_1^1, e_1^2) \ldots, (e_d^1, e_d^2)) = \mu^3(\mathbf{e})$. Let player 3 choose randomly an element of $f_1^3(e_1^1, \ldots, e_d^1)$ and send it to player 2.

P3(1,2) is as follows:

1. 1 selects $(e_1^1, \ldots, e_d^1) \in E^1(\mu^3)$ and sends it privately to 3,

2. 3 publicly announces STOP if $(e_1^1, \ldots, e_d^1) \notin E^1(\mu^3)$,

3. 3 selects $(e_1^2, \ldots, e_d^2) \in f_1^3(e_1^1, \ldots, e_d^1)$ and sends it privately to 2,

4. 2 publicly announces STOP if $(e_1^2, \ldots, e_d^2) \notin E^2(\mu^3)$.

Player 1 cannot deviated without being detected by 3, though player 2 does not know whether the deviator is 1 or 3. 3 can deviate by saying stop though $(e_1^1, \ldots, e_d^1) \in E^1(\mu^3)$. 3 can deviate and send a message from the set $E^2(\mu^3)$ but not from the set $f_1^3(e_1^1, \ldots, e_d^1)$. Player 2 can also deviate by saying stop though $(e_1^2, \ldots, e_d^2) \in E^2(\mu^3)$ and player 1 does not know the identity of the deviator.

Construct P3(2,1) similarly by changing the roles of 1 and 2.

7.2 Random Bcc messages

We construct a protocol to avoid the possible deviations mentioned above. In case of deviation we also want that the identity of the deviator is revealed with high probability. We build a protocol to generate the information structure μ^3.

Definition 7 *A message from player 1 is* $\mathrm{Bcc}(2)$ *to player 3 if player 2 gets the message as well, however 3 does not know if 2 got it or not.*

P3':

Initialization: set $x_1 = x_2 = x_3 = 0$,

if there exists an $i \in \{1, 2, 3\}$ such that $x_i > x_j$ for all $j \neq i$, then i is identified as a deviator and P3' STOPs.

P3'(1,2):

1. 1 selects $(e_1^1, \ldots, e_d^1) \in E^1(\mu^3)$ and sends it privately or Bcc(2) to 3 with probability $(1 - q), q$ respectively,

2. 3 publicly announces STOP if $(e_1^1, \ldots, e_d^1) \notin E^1(\mu^3)$.

3. 3 selects $(e_1^2, \ldots, e_d^2) \in f_1^3(e_1^1, \ldots, e_d^1)$ and sends it privately or Bcc(1) to 2 with probability $(1 - q), q$ respectively, to 2,

4. 2 publicly announces STOP if

    (a) $(e_1^2, \ldots, e_d^2) \notin E^2(\mu^3)$,

    (b) at stage 1 the message was Bcc(2) and $(e_1^2, \ldots, e_d^2) \notin f_1^3(e_1^1, \ldots, e_d^1)$.

5. Player 1, 2 and 3 announces publicly if there were Bcc messages or not in stage 1 and 3.

    (a) In case the concordant messages are NO goto stage 6.

    (b) In case of contradicting messages:

        i. if 1 says NO and 2 YES then STOP.
        ii. if 1 says YES and 2 NO then set $x_1 = x_1 + 1$ and $x_2 = x_2 + 1$ and STOP.
        iii. if 1 says NO and 3 YES then set $x_1 = x_1 + 1$ and $x_3 = x_3 + 1$ and STOP.
        iv. if says 1 YES and 3 NO then STOP.
        v. if 1 contradicts with 2 and 3, 1 is identified as a deviator and P3' STOPs.

    (c) otherwise goto stage 1.

6. P3' STOPS.

In case of STOP in stages different from 6 and 5(b)v.:

1. in stage 2: 3 is identified as a deviator by 1 and 2 or 1 is identified as a deviator by 2 and 3 with probability q. Otherwise set $x_1 = x_1 + 1$ and $x_3 = x_3 + 1$ and goto P3'(2,1).

2. in stage 4: 3 is identified as a deviator by 1 and 2 or 2 is identified as a deviator by 1 and 3 with probability q. Otherwise set $x_2 = x_2 + 1$ and $x_3 = x_3 + 1$ and goto P3'(2,1).

3. in stage 5 case b i.: 2 announces some $(e_1^1, \ldots, e_d^1)'$. If it is equal with $(e_1^1, \ldots, e_d^1)$ according to player 3, then 1 is identified as a deviator. If not 2 is identified as a deviator.

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4. in stage 7 case b iv.: 1 announces some $(e_1^2, \ldots, e_d^2)'$. If it is equal with $(e_1^2, \ldots, e_d^2)$ according to player 2, then 3 is identified as a deviator. If not 1 is identified as a deviator.

5. in case ii. and iii.: goto P3'(2,1).

In case a deviator is identified P3' STOPs.

Lemma 3 *1. P3' stops in finite time with probability 1 even if one of the players deviates from the protocol.*

2. If a player deviates then it is identified with probability q or the P3' stops at stage 6.

3. If P3' stops at stage 6, player 1 knows not more than $(e_1^1, \ldots, e_d^1) \in E^1(\mu^3)$, player 2 knows not more than $(e_1^2, \ldots, e_d^2) \in f_1^3(e_1^1, \ldots, e_d^1)$ and player 3 knows $(e_1^1, e_1^2) \ldots, (e_d^1, e_d^2)$ or by changing the indexes: 2 knows not more than $(e_1^2, \ldots, e_d^2) \in E^2(\mu^3)$, player 1 knows not more than $(e_1^1, \ldots, e_d^1) \in f_2^3(e_1^2, \ldots, e_d^2)$ and player 3 knows $(e_1^1, e_1^2) \ldots, (e_d^1, e_d^2)$.

Proof: It is straightforward by construction. ∎

Suppose now that P3' stops at stage 6. Let player 1 and 2 privately conduct a jointly controlled lottery on $\{1, \ldots, d\}$ and select a d^* uniformly. That is 1 and 2 selects an $e^* = (e_{d^*}^1, e_{d^*}^2)$ according to $\mu^3(\mathbf{e})$ and 1 learns $e_{d^*}^1$ and believes $\mathbf{e}^2$ with probability $\mu^3(\mathbf{e}^2 | e_{d^*}^1)$, 2 learns $e_{d^*}^2$ and believes $\mathbf{e}^1$ with probability $\mu^3(\mathbf{e}^1 | e_{d^*}^2)$ and player 3 believes $\mathbf{e}$ with probability $\mu^3(\mathbf{e})$.

Lemma 4 *P3' followed by the jointly controlled lottery generates the information structure μ^3 on E_3 in a way that 3 believes the outcome with μ^3 or a deviator is identified with probability q.*

Proof: It is straightforward by construction. ∎

7.3 From 2 players to 3 players

Now take any finite 3 player game Γ and a correlated equilibrium $\mu \in \Delta(A^1 \times A^2 \times A^3)$ with rational entries. Consider the following extended game Γ^c:

1. Players can run P3',

2. players send private and public messages to the others,

3. players choose an action in Γ.

We show that there is a Nash equilibrium strategy profile of Γ^c which generates the distribution $\mu \in \Delta A$.

<div align="center">STEP 1</div>

First, players run the protocol P3'. We have to specify E_3 and μ^3.

Let $L^1 = L^2$, $L^1 \times L^2 = L$ and $\mathcal{L}^3 : L \to A^3$ such that for any $a^3 \in A^3$ and for any $l^1, l^2 \in L^1$

$$\frac{|\{l'^2 \in L^2 | \mathcal{L}^3(l^1, l'^2) = a^3\}|}{|L^2|} = \frac{|\{l'^1 \in L^1 | \mathcal{L}^3(l'^1, l^2) = a^3\}|}{|L^1|} = \mu(a^3)$$

such an $\mathcal{L}^3$ function can be easily constructed with a help of a Latin square.

μ^3 works in the same way as the correlation device in the Proof of Theorem 3. The only differences is that the actions $(a^1, a^2)_{\eta(l)}$ are selected according to $q(l) = \mu(\mathbf{a}^1, \mathbf{a}^2 | \mathcal{L}^3(l))$ for all $l \in L$.

Assume that no deviator was detected and P3' stopped at stage 6. Player 1 and 2 choose $d^* \in \{1, \ldots, d\}$ by conducting a jointly controlled lottery in private talk. Thus

$$e^i_{d*} = (\eta^i, (\phi^{-i}_{\eta(l)}(a^{-i}_{\eta(l)}))_{l \in L}, (\phi^i_{\eta(l)})_{l \in L}).$$

player i's private information for $i = 1, 2$.

<div align="center">STEP 2</div>

Let player 1 and 2 choose randomly $l^1, l^2 \in L^1$. In equilibrium Player 3's action will be $\mathcal{L}^3(l^1, l^2)$. Notice that by the property of $\mathcal{L}^3$ player 1 or 2 cannot modify the randomness of the selection. Moreover, l^i does not give any information about $\mathcal{L}^3(l^1, l^2)$.

Let for $i = 1, 2$, player i of "type" l^i make the public announcement $\eta^i(l^i)$.

Now for example, player 1 could send $\phi^2_{\eta(l)}(a^2_{\eta(l)})$ to player 2 and so 2 could compute $a^2_{\eta(l)} = \phi^{2-1}_{\eta(l)}(\phi^2_{\eta(l)}(a^2_{\eta(l)}))$ and take this action. The same is true for player 1 if 2 sends him the right information. Notice again that $(a^1_{\eta(l)}, a^2_{\eta(l)})$ was chosen according to $q(l) = \mu(\mathbf{a}^1, \mathbf{a}^2 | \mathcal{L}^3(l))$.

The problem is that 1 or 2 could deviate and send a wrong message to the other. To avoid such deviations in STEP 2, μ^3 selects codes and code functions as well just as in the proof of Theorem 3 or as in Forges (1990).

Let μ^3 select randomly from a set C^i codes for all $(l, a^i) \in L \times A^i$ that is, $c^i(\eta(l), a^i) \in C^i$ and send it to player i for $i = 1, 2$. μ^3 sends to player $-i$ only just the codes corresponding

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to the pairs $(\eta(l), \phi^i_{\eta(l)}(a^i_{\eta(l)}))$ that is, the codes $(c^i(\eta(l), \phi^i_{\eta(l)}(a^i_{\eta(l)})))_{l \in L}$.

Now player 1 and 2 are able to detect if 1 or 2 sends a wrong message to the other, however player 3 will not know who is the deviator. For this reason we introduce an additional step:

<div align="center">STEP 3</div>

Player 1 and 2 send:

1. $c^i()$ the code functions,

2. $c^i(\eta(l), \phi^i_{\eta(l)}(a^i_{\eta(l)}))$, the codes,

3. $\phi^i_{\eta(l)}(a^i_{\eta(l)})$ the relevant information

privately to player 3. Now player 3 can check if there is a contradiction among the codes, the code functions and relevant information and announce STOP in this case. Notice that player 3 can deviate and stop the process though 1 and 2 sent the correct messages.

Up to this point none of the players knows anything about his or the others' action.

If player 3 announces STOP the players proceed with STEP 6, otherwise they go to STEP 4.

<div align="center">STEP 4</div>

Player 1 and 2 sends l^1, l^2 to player 3 and he can compute his action $\mathcal{L}^3(l)$. 1 and 2 has no incentives to deviate. Their situation is just as in a communication equilibrium.[19]

<div align="center">STEP 5</div>

Now player 1 sends to player 2 the relevant information and the code. Player 2 does the same for player 1, and so 1 and 2 can compute their own actions and check if the other was deviating or not.

In case of deviation player 3 is decisive (knows the necessary information) and the deviator is minmaxed.

<div align="center">STEP 6</div>

In case of a stop announcement in STEP 3 the players announce publicly the messages sent in STEP 3. A pair of players can be identified one of which is the deviator. That is some player j can be identified who was not deviating for sure. Let player j select an action profile according to μ and inform $-j$ about their own actions. Players different from j have incentives to follow the suggestion. Q.E.D

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[19]By deviating player 1 or 2 can expect less than in μ.

8 Discussion

Several articles investigate how to circumvent the necessity of the mediator or, at least, in some sense minimize his duties, knowledge of the situation and influence on the outcome as the game or the implemented distribution varies. We just mention a few of them which motivated our investigation.

8.1 Two player case

An important result is due to Urbano and Vila (2002). Two players can solve the CE implementation problem without any mediation if certain calculations are hard to perform that is, players are computationally restricted. In case of games with complete information, assuming full rationality and plain conversation players cannot surpass the convex hull of Nash equilibrium payoffs. If players cannot use urns or envelopes as in Krishna (2004), welfare improvement is impossible without a third party.

Lehrer (1996), Lehrer and Sorin (1997) and Vida (2003) show how the players can replace fortune and avoid the use of private messages from the mediator. That is, the mediator's task is to make public announcements, which are deterministic functions of the players' private messages. However, in all solutions proposed in these papers, the mediator has to be tailor-made to the particular game and CE at hand.

In Theorem 1 we present a unique, simple, deterministic mediator which can be applied in any finite game without loss of efficiency.

One can attach various interpretations to the *AND* device. For example players may look or may not look in each others' face during their conversation. The mimicry and its observation is clearly important part of human communication. Imagine the following situation. An internet community offers to its members the possibility of establishing a link[20]. Each member has the possibility to accept or reject the link formation. The link forms if and only if both parties yield consent to it. Both examples produce the *AND* signalling structure. It is clear that the *AND* includes all the necessary components to generate any information structure. The intuition is that players have the possibility to go through a check list and avoid undesired outcomes. However, to achieve incentive compatibility the *AND* is not enough in itself. Another coordination problem arises. The main idea in the proof of Theorem 1 is that players need to talk directly to be able to coordinate their actions accordingly. Nevertheless, we cannot guarantee Nash equilibrium in general. The equilibrium is subject to an ϵ. It is mainly because of the finiteness of the protocol, the necessity of independent randomization and the flexibility of the mechanism. It is interesting to mention the connection here with the finitely repeated game literature. For example in Gossner (1995) the mixed strategies are not observable just their realizations.

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[20]For reasons of common interests, common friends etc..

Players need to test whether others follow the equilibrium punishment strategies or not. The test needs to have the property that whenever players pass the test the punishment is effective. In Proposition 4 we had the same situation. If players pass the test the generated distribution has to be close to the desired one.

In Bayesian games, plain conversation itself can be welfare improving. Aumann and Hart (2003) characterize the payoffs achievable by plain conversation in terms of bi-martingales, where only one of the players has private information. Amitai (1996) solves the two sided incomplete information case by characterizing the martingales leading to equilibrium payoffs. Interestingly, in this case the equilibrium payoffs depend on the possible messages available for the players. He also considers the cases of polite talk and talks with random stopping. Urbano and Vila (2004) implements the whole set of communication equilibria of games with computationally restricted players.

Our contribution here is in Theorem 3 and Corollary 1. Using the *AND* players can implement any Nash dominant communication equilibrium.

8.2   More than two players

If the number of players is at least 4 there is no need of mediation at all. Barany (1992) shows that, if the sent messages can be recorded and publicly checked then the players can implement any CE in Nash equilibrium of a cheap talk extended game, where min-max strategies are used as punishments in case of deviation from the prescribed strategies. Gerardi (2000) extends Barany's protocol and shows that there is no need of punishment at all. In case of 5 players even sequential rationality is maintainable.

If the number of players is at least 4, one can generate any correlated distribution following Barany's (1992) protocol. Forges (1990) uses this protocol to implement the set of communication equilibria in Nash equilibria of an extended game. However, in her set up there is communication in ex ante stages. Gerardi (2000) extends Barany's protocol and achieves the same result as Forges (1990) only with interim stages of communication. Each of these protocols assume that past messages were recorded and can be publicly checked.

Ben-Porath (2003) deals with the case of 3 player and proves that the strongly Nash dominant communication equilibria are implementable in sequential equilibrium. In fact, in case of deviation in the Ben-Porath's protocol a pair of players is identified, one of which is the deviator. The main difficulty is that deviations can be beneficial for the players and need to be punished.

Our contribution here is in Theorem 2 and Corollary 2. The assumption that players can send blind carbon copies of their messages, allows us to construct a protocol in which a deviator is identified with arbitrary high probability. Thus our implemented set is bigger than that of Ben-Porath. However, we cannot maintain sequential rationality.

Our Theorems are intuitively connected in the following way. In Theorem 1 we learn how to build any information structure for two players in a way that the mediator's malfunctioning can be tested by the players and the mediator gets no knowledge about the outcome of the correlation. That is, in a three player setup players can generate any information structure for players 1 and 2 in a way that player 3 takes the role of the mediator. It then turns out that the players' action can be constructed as player 1 and 2 communicate in a game with incomplete information just as in Theorem 2. Finally, 1 and 2 inform player 3 about their "types", which then defines the action of player 3.

8.3 Refinements

There is another branch of the cheap talk literature which concentrates on refining equilibria and not on enlarging the set of equilibrium payoffs as the ones above. For a good survey see Farrel and Rabin (1996). Messages can have some intrinsic meaning. For example in Farrell (1988) players are offering to play one of the Nash equilibria. In our paper, during the mediated phase the players are basically offering actions to play. Bad action profiles are ruled and sorted out by the *AND*. We can think of the communication as a negotiation process for an acceptable self enforcing distribution. An interesting question is how costly communication processes affect the equilibrium outcomes.

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